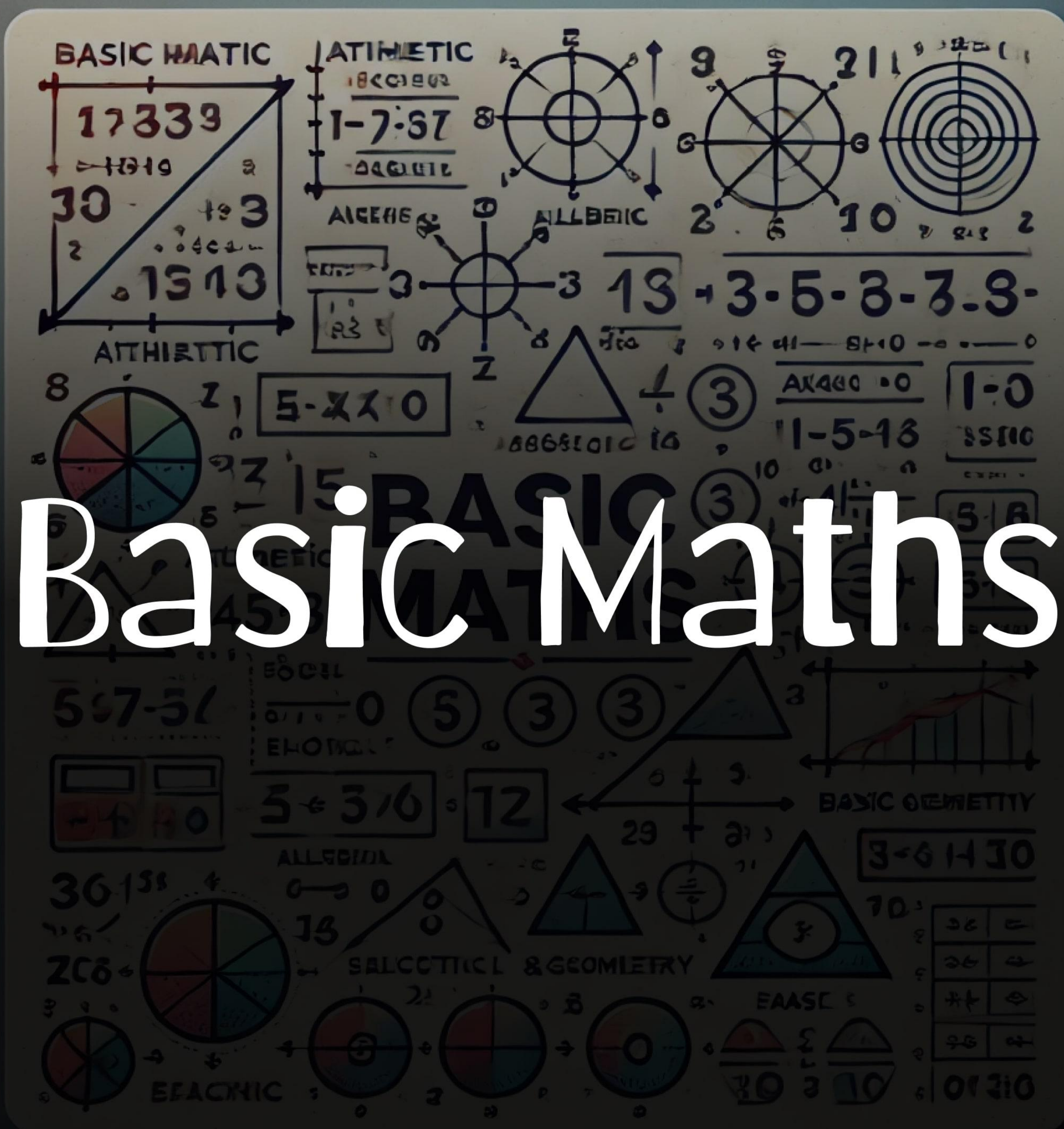
Basic Maths



Basic Mathematics

Rules of Power (Exponent)

1. If power of any non-zero no. is zero then result will be 1

$8^0 = 1$ $\qquad$ $10^0 = 1$

$e^0 = (2.71)^0 = 1$

2. Negative Property of exponent (x is non-zero no.)

$x^n = \frac{1}{x^{-n}}$ $\qquad$ $\frac{1}{x^n} = x^{-n}$

$5^3 = \frac{1}{(5^{-3})}$ $\qquad$ $\frac{1}{10^2} = 10^{-2}$

3. Product Property of Exponent

$x^n x^m = x^{n+m}$

$10^3 \times 10^4 = 10^{3+4} = 10^7$

4. Division Property of Exponent

$\frac{x^n}{x^m} = x^{n-m}$

$\frac{10^7}{10^{-4}} = 10^7 \times 10^4 = 10^{11}$

5. Power of a Power

$(x^n)^m = x^{n \times m}$

$(10^2)^3 = 10^{2\times 3} = 10^6$

6. Fractional Exponent

$(x)^{3/2} = (x^3)^{1/2} = \sqrt[2]{x^3}$

$(9)^{3/2} = [(9)^{1/2}]^3 \Rightarrow [\sqrt{9}]^3 \Rightarrow (3)^3 = 27$

Multiplication and Division

$0.5 = \frac{5}{10} = \frac{1}{2}$

$0.75 = \frac{75}{100} = \frac{3}{4}$

$0.6 = \frac{6}{10} = \frac{3}{5}$

$0.33 = \frac{1}{3}$

$0.4 = \frac{4}{10} = \frac{2}{5}$

$0.64 = \frac{64}{100} = \frac{16}{25}$

$0.66 = \frac{2}{3}$

$0.49 = \frac{49}{100}$

$1.25 = \frac{125}{100} = \frac{5}{4}$

$0.25 = \frac{25}{100}$

$1.33 = \frac{4}{3}$

What is Infinity?

Is one divided by zero is infinite ??

$\frac{1}{0} = \infty =$ undefine

$e^{\infty} =$ infinite

$\frac{1}{\infty} = 0$

$e^{-\infty} = 0$

$2^{\infty} =$ infinite

$2^{-\infty} = \frac{1}{2^{\infty}} = \frac{1}{\text{infinite}} = 0$

Concept of log

Power

$2 \times 2 \times 2 = (2)^3 = 8 \rightarrow$ Result

Base

Same concept

Result

$\log (2)^8 = 3 \rightarrow$ Power

Base

$\log_{10}^{100} = x$

Antilog

$10^x = 100$

$x = 2$

$\log_2^{16} = 4$

↓

log(16) on the base(2)

→ Same as concept of power

$2^4 = 16$

$1\,pC = 10^{-12}C$ (Pico)

$1\,nC = 10^{-9}C$ (nano)

$1\,\mu F = 10^{-6}f$ (micro)

$1\,mm = 10^{-3}m$ (milli)

$1\,KJ = 10^3 J$ (kilo)

$1\,MW = 10^6 W$ (Mega)

$1\,GN = 10^9 N$ (Giga)

Concept of log

Base → remains there and interchange Power & Result

$(4)^3 = 64$ → Result

Base

Power

$\log_4^{64} = 3$

Antilog → $4^3 = 64$

Property - 01

$\log_a^1 = 0$

$\log_e^1 = 0$

$\log_{10}^1 = 0$

$\log_5^1 = 0$

('a' may be value but not equal to 0 & 1)

→ $5^0 = 1$ → $\log_5^1 = 0$

$\log_{10}^{\sin 90^\circ} = 0$; $\log_e^{(\sin^2\theta + \cos^2\theta)} = 0$; $\log_e^{(\tan\theta \cdot \cot\theta)} = 0$

Property - 02

$\log_a^a = 1$ $\log_5^5 = 1$

$\log_e^e = 1$ $\log_2^2 = 1$

$\log_{10}^{10} = 1$ (Power) $\log_3^3 = 1$

$\log_y^x$ = log 'x' on the base 'y'

$\log_y^{x^n}$ = log 'x' to the power n on the base 'y'

$\log_{(y^n)}^x$ = log 'x' on the base 'y' to the power n

Logarithmic

Natural Log

($\log_e^x = \ln x$)

log 'x' on the base 'e'

Common Log

($\log_{10}^x$)

MR Ratta

$$\left(\log_e^x = 2.303 \log_{10}^x\right)$$

$$(\log_{10}^2 = 0.3010 \approx 0.30)$$

$$(\log_{10}^3 = 0.4771 \approx 0.48 \approx 0.5)$$

Property - 03

$\log_y^{x^n} = n \log_y^x$ → Example - $\log_{10}^{100} \Rightarrow \log_{10}^{(10)^2}$

$$= 2\log_{10}^{10}$$

$$= 2$$

$\log_{(y)^n} x = \frac{1}{n}\log_y x \rightarrow$ Example - $\log_{100} 10 \Rightarrow \log_{(10)^2} 10$

$= \frac{1}{2}\log_{10} 10 \Rightarrow \frac{1}{2}$

$\log_e (xy) = \log_e x + \log_e y$

$\log_e \left(\frac{x}{y}\right) = \log_e x - \log_e y$

Special case

$\log_y x = \frac{1}{\log_x y}$

$\log_y x = \frac{\log_{10} x}{\log_{10} y}$

$\log_b a \times \log_a b = 1$

$\log_y x = \frac{\log_e x}{\log_e y}$

Que-

(i) $\log_{10} 6 = \log_{10} (3\times2) \Rightarrow \log_{10} 3 + \log_{10} 2 \Rightarrow 0.48 + 0.30 = 0.78$

(ii) $\log_{10} 5 = \log_{10}\left(\frac{10}{2}\right) \Rightarrow \log_{10} 10 - \log_{10} 2 \Rightarrow 1 - 0.30 = 0.7$

(iii) $\log_e 10 = 2.303 \times \log_{10} 10 \Rightarrow 2.303$

(iv) $\log_2 10 = \frac{1}{\log_{10} 2} \Rightarrow \frac{1}{0.30} \Rightarrow \frac{10}{3} = 3.33$

(v) $\log_8 16 = \log_{(2)^3} (2)^4 \Rightarrow \frac{4}{3}\log_2 2 = \frac{4}{3}$

(vi) $\log_{27} 3 = \log_{(3)^3} 3 \Rightarrow \frac{1}{3}\log_3 3 = \frac{1}{3}$

(vii) $\log_{(0.01)} 10 = \log_{10^{-2}} 10 \Rightarrow -\frac{1}{2}\log_{10} 10 = -\frac{1}{2}$

(viii) $\log_{10} 0.001 = \log_{10}\left(\frac{1}{1000}\right) \Rightarrow \log_{10} 10^{-3} \Rightarrow -3\log_{10} 10 = -3$

Que-

(i) $\log_{10}5 + \log_{10}20 = \log_{10}{20\times5} \Rightarrow \log_{10}100 = 2$

(ii) $\log_{10}60 - \log_{10}6 = \log_{10}\left(\frac{60}{6}\right) \Rightarrow \log_{10}10 = 1$

(iii) $\log_2(1/8) = \log_2 2^{-3} \Rightarrow -3\log_2 2 = -3$

(iv) $\log_2 3 = \dfrac{\log_{10}3}{\log_{10}2} \Rightarrow \dfrac{0.48}{0.30} = \dfrac{48}{30}$

$\log_e 2 = 2.303 \times \log_{10}2 \Rightarrow 2.303 \times 0.3010 \Rightarrow 0.693$

Concept of Antilog

$\log_e x = y$

By taking anti-log (convert it into concept of power)

$e^y = x$

$\log_e x = y$

Que- Find value of given expression

$\log_{10}(4\times10^{-4}) \Rightarrow \log_{10}4 + \log_{10}10^{-4}$

$= 2\log_{10}2 - 4\log_{10}10$

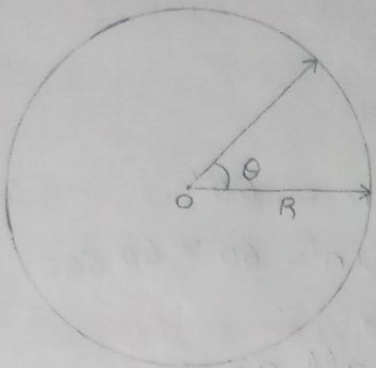
$= 0.6 - 4$

$\Rightarrow -3.4$

Trigonometry

Angle (θ) → algebraic function (x, t, v, θ)

$\sin\theta, \cos\theta, \tan\theta$ → Trigonometric Ratio



Arc $\propto$ Angle (θ)
Arc $\propto$ Radius (R) → $\boxed{Arc = R\theta}$

Plane Angle ← $\boxed{\theta = \dfrac{Arc}{Radius}}$ = $\dfrac{metre}{metre}$

Que- Is angle is unitless?
Ans- No, angle have unit - Radian (S.I unit)

→ Angle have unit, but does not have dimension.
→ Supplementary Physical quantity.

→ S.I unit or CGS unit → Radian

→ Other unit (Practical unit) → Degree, minute, second

For Algebraic Function (θ) we always use radian

For Trigonometric ratio we can use both degree or Radian.

Relation between degree and Radian

(MR Ratta) → $180° = \pi$ radian → Radian is a bigger unit

$\boxed{1° = \dfrac{\pi}{180°} \text{ rad}}$ $\boxed{1 \text{ rad} = \dfrac{180°}{\pi}}$

Q- Convert following radian into degree

(i) $\dfrac{\pi}{2}$ rad $= \dfrac{\pi}{2} \times \dfrac{180}{\pi} \Rightarrow \dfrac{180}{2} \Rightarrow 90°$

MR* = consider π rad $= 180°$ (directly)

Que- convert following degree into radian

(i) $45° = 45 \times \frac{\pi \text{ rad}}{180°} \Rightarrow \frac{\pi}{4}$ rad

(ii) $\pi° = \pi \times \frac{\pi \text{ rad}}{180°} \Rightarrow \frac{\pi^2}{180}$ rad

Que- convert 1 radian into minute

$\theta = 1$ rad

$= \frac{180°}{\pi}$

$1° = 60 \text{ min} = 60'$

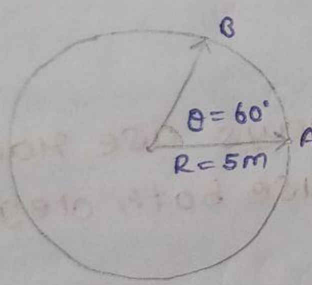
$1° = 60 \text{ min} = 60 \times 60$ sec

for small angle

$\theta = \frac{180 \times 1°}{\pi} = \frac{180}{\pi} \times 60$ min

$1 \text{ rad} = \frac{180 \times 60}{\pi}$ min

Que- when object moves 60° on circular path of radius 5 m then distance travelled by object



Distance = Arc = $R\theta$ ← rad

$= 5 \times 60 = 300$ m (wrong)

$5 \times 60 \Rightarrow 5 \times 60 \times \frac{\pi}{180}$ rad

Distance $\Rightarrow \frac{5\pi}{3}$ m (Right)

Que- Find total angle moved by object in π rotation

Angle in one rotation = 2π rad

" π rotation = $(2\pi)\pi$ rad

$= 2\pi^2$ rad

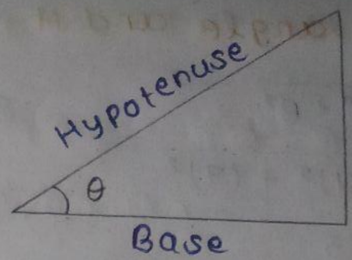
π rad = Angle

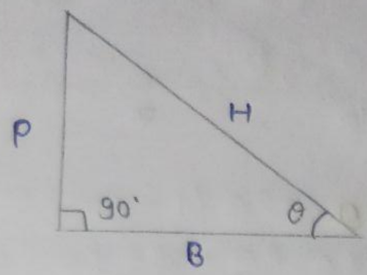
π Newton = Force

π Joule = Energy

Trigonometric Ratio

* $H_2 = P^2 + B^2$





$\sin\theta = \frac{P}{H}$; $\frac{H}{P} = \operatorname{cosec}\theta$

$\cos\theta = \frac{B}{H}$; $\frac{H}{B} = \sec\theta$

$\tan\theta = \frac{P}{B}$; $\frac{B}{P} = \cot\theta$

$\cos\theta \cdot \sec\theta = 1$

$\tan\theta \cdot \cot\theta = 1$

$\sin\theta \cdot \operatorname{cosec}\theta = 1$

	0°	30°	45°	60	90°	120°	135	150	180
$\sin\theta$	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
$\cos\theta$	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0	$-1/2$	$-1/\sqrt{2}$	$-\sqrt{3}/2$	-1
$\tan\theta$	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-1/\sqrt{3}$	0

$\theta \to 0°$ to $90°$

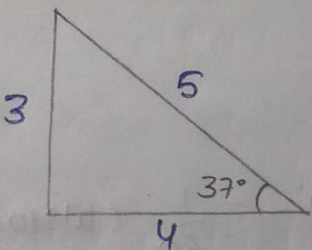
$\sin\theta \uparrow$
$\cos\theta \downarrow$ $\rightarrow$ $\tan\theta = \frac{\sin\theta \uparrow}{\cos\theta \downarrow} = \uparrow$

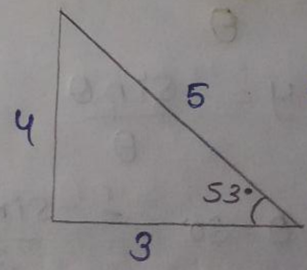
Special Angle

$\sin 37° = \frac{3}{5}$

$\cos 37° = \frac{4}{5}$

$\tan 37° = \frac{3}{4}$





$\sin 53° = \frac{4}{5}$

$\cos 53° = \frac{3}{5}$

$\tan 53° = \frac{4}{3}$

Que- Find angle and H

$H = \sqrt{P^2 + B^2}$

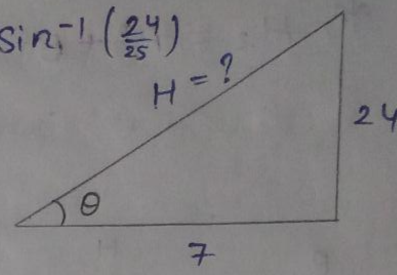
$H = \sqrt{(24)^2 + (7)^2}$

$H = \sqrt{625}$

$H = 25$

$\sin\theta = \frac{24}{25} \rightarrow \theta = \sin^{-1}\left(\frac{24}{25}\right)$

$\tan\theta = \frac{24}{7}$



Que- If $\tan\theta = 2$ then find $\sin\theta$ and $\cos\theta$

$\tan\theta = 2 = \frac{P}{B} = 2$, If I assume $P = 2$ & $B = 1$

$H = \sqrt{P^2 + B^2} = \sqrt{4+1} = \sqrt{5}$

$\sin\theta = \frac{P}{H} = \frac{2}{\sqrt{5}}$

$\cos\theta = \frac{1}{\sqrt{5}}$

Que- If value of $\sin\theta = \frac{4}{3}$ then find value of $\cos\theta$ and $\tan\theta$

$\sin\theta = \frac{P}{H} = \frac{4}{3}$

$\boxed{-1 \le \sin\theta \le 1}$

$\sin 270^\circ = -1$

$\sin\theta$ can't be greater than 1

$\sin\theta_{max} = 1$

$\sin\theta_{min} = -1$

Hypotenuse > Perpendicular (always)

Que- If $y = \frac{\sin\theta}{\theta}$ then find value of y if $\theta = 30^\circ$.

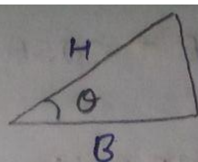
$y = \frac{\sin\theta}{\theta}$

$y_{at\ \theta=30^\circ} = \frac{\sin 30^\circ}{30} = \frac{1}{60}$ (wrong)

$\frac{\sin 30^\circ}{30} \Rightarrow \frac{1/2}{\pi/6} \Rightarrow \frac{6}{2\pi} = \frac{3}{\pi}$

$(P^2 + B^2 = H^2)$

If divided by P^2 both sides



$$\frac{P^2}{P^2} + \frac{B^2}{P^2} = \frac{H^2}{P^2}$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$P^2 + B^2 = H^2$

divided by H^2 both Side

$$\frac{P^2}{H^2} + \frac{B^2}{H^2} = \frac{H^2}{H^2}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin\theta = \sqrt{1-\cos^2\theta}$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

Trigonometric Identities

(i) $\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

(ii) $\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$

(iii) $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

(iv) $\cos(A-B) = \cos A \cdot \cos B + \sin A \cdot \sin B$

(v) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

Que- $\sin(\underset{\uparrow A}{90} + \underset{\uparrow B}{\theta}) = \sin 90^\circ \cdot \cos\theta + \overset{0}{\cos 90^\circ} \cdot \sin\theta$

$= 1\cos\theta + 0$

$\sin(90+\theta) = \cos\theta$

$\cos(330^\circ) = \cos(360 - 30)$

$= \cos(360^\circ) \cdot \cos(30^\circ) + \sin 360^\circ \cdot \sin 30^\circ$

$= 1 \times \frac{\sqrt{3}}{2} + 0 \Rightarrow \frac{\sqrt{3}}{2}$

Que- If $A = B$ then $\sin(A+A)$

$\sin A \cdot \cos A + \cos A \cdot \sin A$

$\sin(2A) = 2\sin A \cdot \cos A$

$\sin(2\theta) = 2\sin\theta \cdot \cos\theta$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$\downarrow$

If $[A = B = \theta]$ MR Ratta

$\boxed{\sin(2\theta) = 2\sin\theta \cdot \cos\theta}$

$\downarrow$

If $[2\theta = \alpha]$

$\sin\alpha = 2\sin(\alpha/2) \cdot \cos(\alpha/2)$

Half angle formula

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$\downarrow$

If $[A = B = \theta]$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos\theta = \cos^2\theta/2 - \sin^2\theta/2$

Half Angle

$\cos(2\theta) = \cos^2\theta(1 - \cos^2\theta)$

$= \cos^2\theta - 1 + \cos^2\theta$

$\cos 2\theta = 2\cos^2\theta - 1$

$2\cos^2\theta = 1 + \cos(2\theta)$

Trigonometric function change

$\sin\theta$, $\operatorname{cosec}\theta$ +ve	All +ve
$\tan\theta$, $\cot\theta$ +ve	$\cos\theta$, $\sec\theta$ +ve

90° $(\frac{\pi}{2})$ rad; 180° (π rad); $0^\circ/360^\circ$ (2π rad); 270° $\frac{3\pi}{2}$

$\sin(90-\theta) = \cos\theta$

$\cos(90-\theta) = \sin\theta$

$\sin(90+\theta) = \cos\theta$

$\sin(180-\theta) = \sin\theta$

$\cos(180-\theta) = -\cos\theta$

$\sin(0^\circ) = \sin(360^\circ) = 0$

$\cos(0^\circ) = \cos(360^\circ) = 1$

$\cos(180-\theta) = -\cos\theta$

$\cos(270-\theta) = -\sin\theta$

Que - Find value of

(i) $\sin(-30^\circ) = \sin(0-30^\circ) = \sin 0^\circ \cos 30^\circ - \cos 0^\circ \sin 30^\circ$

$= -\sin 30^\circ \Rightarrow -1/2$

(ii) $\cos(-60^\circ) = \cos(0^\circ - 60^\circ) = \cos 0^\circ \cos 60^\circ + \sin 0^\circ \sin 60^\circ$
$= \cos 60^\circ \Rightarrow 1/2$

(iii) $\tan(-30^\circ) = \tan(0-30^\circ) = \dfrac{\tan A - \tan B}{1+\tan A \cdot \tan B} = \dfrac{\tan 0^\circ - \tan 30^\circ}{1+\tan 0^\circ + \tan 30^\circ}$
$= \dfrac{-\tan 30^\circ}{1} = \tan 30^\circ$

Odd function

$\sin(-\theta) = -\sin\theta$

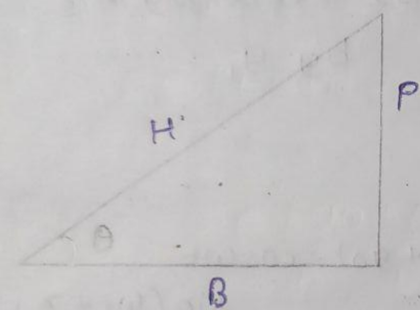
$\cos(-\theta) = \cos\theta \leftarrow$ Even function

$\tan(-\theta) = -\tan\theta$

$\cos(-30^\circ) = \cos 30^\circ = \sqrt{3}/2$ $\quad\tan(-45^\circ) = -\tan 45^\circ = -1$

$\sin(-30^\circ) = -\sin 30^\circ = -1/2$

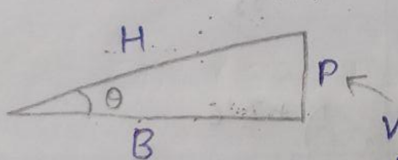
Small Angle approximation



$\cos\theta = \dfrac{B}{H}$ If θ is very small
$[\theta \leq 7^\circ]$
$B = H$
$\cos\theta = \dfrac{\cancel{B}}{\cancel{H}} = 1$

$\cos 1^\circ = 1$
$\cos 2^\circ = 1$
$\cos 4^\circ = 1$
$\cos 5^\circ = 1$

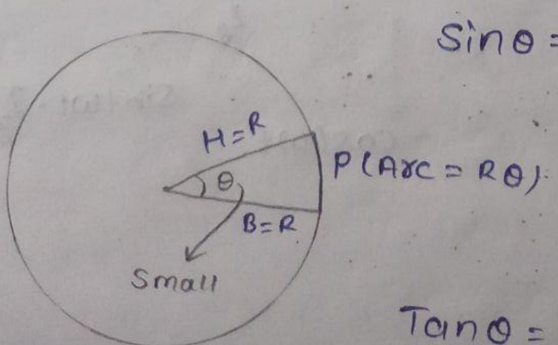
$\sec 3^\circ = 1$
$\sec 3^\circ = \dfrac{1}{\cos 3^\circ} = \dfrac{1}{1}$



very small

$H^2 = P^2 + B^2$

θ_{small} $\dfrac{\sin\theta}{\theta} = \dfrac{\cancel{\theta}}{\cancel{\theta}} = 1$



$\sin\theta = \dfrac{P}{H} = \dfrac{R\theta}{R}$

$\sin\theta = \theta$

Angle is very small

for small angle
$\sin\theta = \tan\theta$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{\theta}{1}$

$\tan\theta = \dfrac{P}{B} = \dfrac{\cancel{R}\theta}{\cancel{R}} = \theta$

What is phase

$y = 4\sin\boxed{(\omega t + \phi)}$

Phase at time t $= (\omega t + \phi)$

$y = 4\sin(\omega t + \phi)$

at $t = 0$

$y = 4\sin(\phi) \rightarrow$ Phase at $t = 0$

Que- Two waves are represented by the equation $y_1 = 4\sin(3t)$ and $y_2 = 4\sin(3t + \pi/2)$. Determine the phase difference b/w the two waves

$y_1 = 4\sin\underbrace{(3t)}_{\theta_1}$ $\quad$ $y_2 = 4\sin\underbrace{(3t + \pi/2)}_{\theta_2}$

Phase difference $= \theta_2 - \theta_1$

$= 3t + \pi/2 - 3t$

$\phi = \pi/2$

y_2 leads by $\pi/2$ by y_1

Phaser Diagram

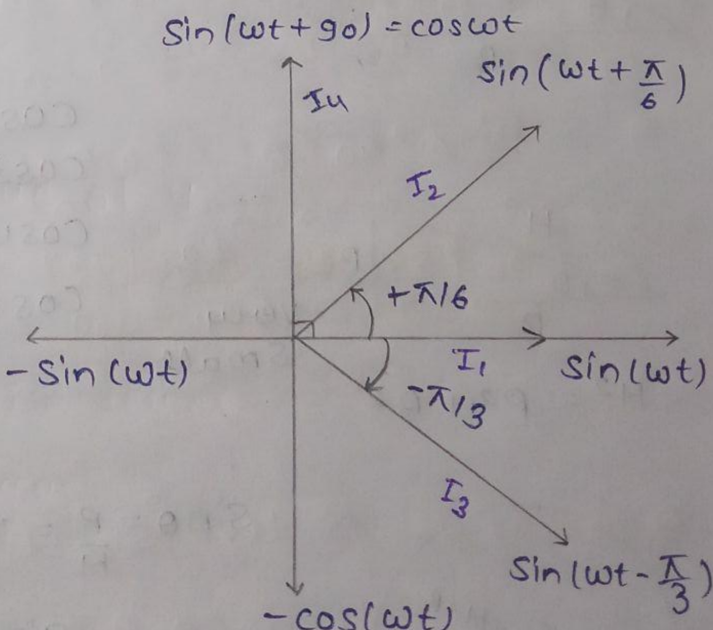
cos θ leads by sin θ through angle $\pi/2$

$I_1 = 10\sin(\omega t)$

$I_2 = 10\sin(\omega t + \frac{\pi}{6})$

$I_3 = 10\sin(\omega t - \pi/3)$

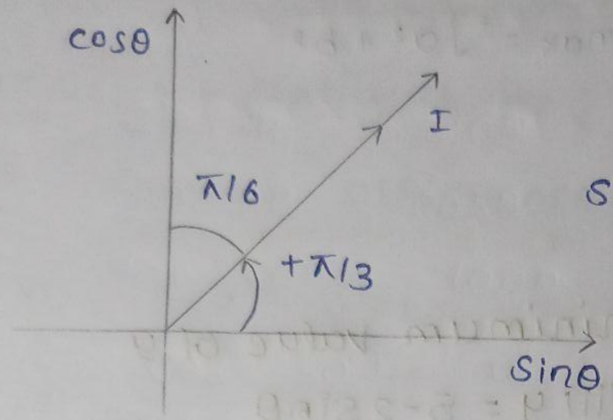
$I_4 = 10\sin(\omega t + 90^\circ)$



Equation of current and voltage

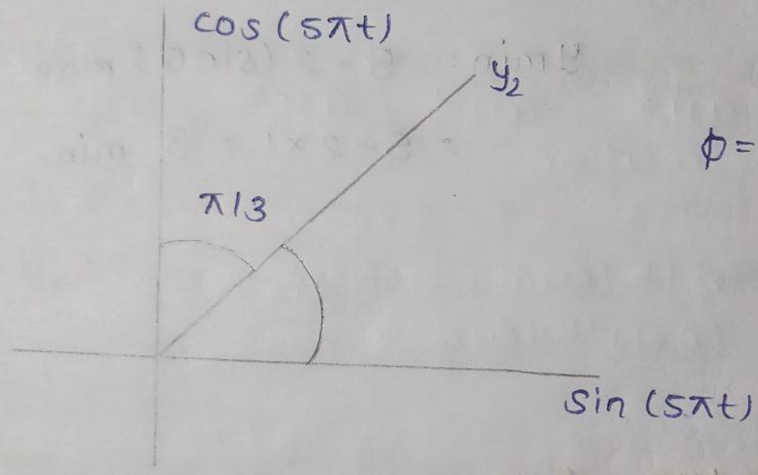
$I = 10 \sin\left(\theta + \frac{\pi}{3}\right)$ and $V = 10 \cos\left(\theta - \frac{\pi}{6}\right)$

then phase difference b/w current & voltage



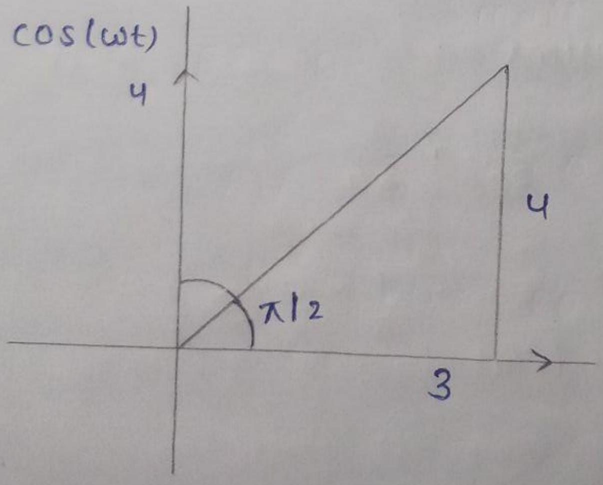
Same phase

Que- If $y_1 = 2\sin(5\pi t)$ and $y_2 = 2\cos(5\pi t - \pi/3)$, what is the phase difference b/w two waveforms



$$\phi = \frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi - 2\pi}{6} = \frac{\pi}{6}$$

Que- If two A/C current $I_1 = 3\sin(\omega t)$ and $I_2 = 4\cos(\omega t)$ then find $I_1 + I_2$



$$I = I_1 + I_2 = 5 \Rightarrow \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

Que- Find maximum and minimum value of y

$$y = 3\sin\theta + 4\cos\theta$$

$y_{max} = \sqrt{(3)^2 + (4)^2}$

$= \sqrt{25}$

$= +5$

$y = \pm a\sin(\omega t) \pm b\cos(\omega t)$

$y_{max} = \pm\sqrt{a^2 + b^2}$

$y_{min} = -5$

Que - Find maximum and minimum value of y

(i) $y = 3\sin(2\theta)$

$y_{max} = 3\sin(2\theta)$

$y_{max} = 3$

at $\theta = 45°$

$2\theta = 90°$

$y_{min} = -3$

at $\theta = 135°$

$2\theta = 270°$

(ii) $y = 5 - 2\sin\theta$

$y_{max} = 5 - 2(\sin\theta)_{min}$

$= 5 - 2(-1) = 7$ max

$y_{min} = 5 - 2(\sin\theta)_{max}$

$= 5 - 2\times 1 = 3$ min.

Que - Find minimum value of y

$$y = \frac{2}{\sqrt{3}\sin\theta + \cos\theta}$$

$$y_{min} = \frac{2}{[\sqrt{3}\sin\theta + \cos\theta]_{max}}$$

$$= \frac{2}{\sqrt{(\sqrt{3})^2 + (1)^2}}$$

$$y_{min} = \frac{2}{\sqrt{3+1}} = \frac{2}{2} = 1$$

Arithmetic Progression

AP series → A sequence of numbers in which difference between any two consecutive number is a constant value.

Example - Natural no. - 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 - - - -

Common Difference = $n^{th} - (n-1)^{th}$

a, $a+d$, $a+2d$, $a+3d$, $a+4d$, $a+5d$ - - - -
↑
1^{st} term

* $d = n^{th} - (n-1)^{th}$

* n^{th} term $= a + (n-1)d$

* sum of n^{th} term $= \frac{n}{2}[2a + (n-1)d]$

" $= \frac{n}{2}[1^{st}\text{ term} + \text{last term or } n^{th}\text{ term}]$

Que - which of the following is not A.P.

1. 2, 8, 15, 21, 27 - - - - - Not A.P
2. 3, 6, 12, 24 - - - - - Not A.P
3. 4, 1, −2, −5, −8 - - - - - A.P.
4. −5, −3, −1, 1 - - - - - A.P.

Que - Find 10th term and sum of 1st 10 term

7, 11, 15, 19 - - - - - - - -

$d = 4$

10^{th} term $= a + (n-1)d$
$= 7 + (10-1) \times 4$
$= 7 + 36$
$= 43$

Sum of 1st ten term

$\text{Sum} = \frac{n}{2}[2a + (n-1)d]$

$\frac{10}{2}[2 \times 7 + (10-1)4]$

$\text{sum} = 5[14 + 36]$
$= 5 \times 50$
$= 250$

Que- Which term of AP 27, 24, 21 - - - - is zero

$a = 27$, $d = 24 - 27 = -3$

n^{th} term $= a + (n-1)d$

$0 = 27 + (n-1) \times (-3)$

$3(n-1) = 27$

$3n - 3 = 27$

$3n = 30$

$n = 10$

Geometric Progression

G.P series - A sequence in which each term is produced by multiplying by preceeding term by a fixed value.

a , ar , ar^2 , ar^3 , ar^4 , ar^5 , ar^6

↑
1st term

r = common ratio $= \dfrac{n\text{th term}}{(n-1)\text{th term}} = \dfrac{ar^4}{ar^3} = r$

$r < 1$ in physics

Sum of Infinite terms $= \boxed{\dfrac{a}{1 - \text{common ratio}}}$

Que- Find sum of infinite terms

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}$ - - - - -

↑
1st term

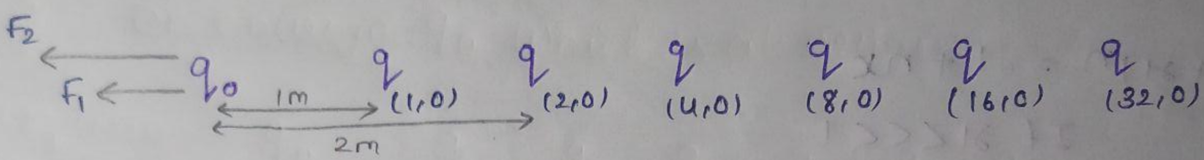
common ratio $= \dfrac{\frac{1}{8}}{(1/4)} = \frac{1}{8} \times \frac{4}{1} \Rightarrow \frac{1}{2}$

$\text{Sum} = \dfrac{a}{1 - \text{c.r.}} = \dfrac{1}{1 - \frac{1}{2}} \Rightarrow \dfrac{1}{1/2} \Rightarrow 2$

Que - 5.- resistance connected in series, resistance of each resistors is half of previous resistance if 1st resistance of value 10 Ω then value of 5th resistance

$R\,(\text{let}),\ \frac{R}{2},\ \frac{R}{4},\ \frac{R}{8},\ \frac{R}{16} \rightarrow \frac{10}{16}\ \Omega$

Que - charge q is placed on x - axis of co-ordinate (1,0), (2,0), (4,0), (8,0) and so on then. Find force on charge q_0 which is at origin



$$F_{net} = \frac{kqq_0}{(1)^2} + \frac{kqq_0}{(2)^2} + \frac{kqq_0}{(4)^2} + \text{-----}$$

$$F_{net} = kqq_0\left(\frac{1}{1} + \frac{1}{4} + \frac{1}{16} + \frac{1}{64} \text{----}\right)$$

$$\text{sum} = \frac{1}{1-\frac{1}{4}} \Rightarrow \frac{1}{\frac{4-1}{4}} \Rightarrow \frac{4}{3}$$

$$F_{net} = \frac{4kqq_0}{3}$$

Que - Find Potential at origin ?

$V = ?$	$+q$	$-q$	$+q$	$-q$	$+q$	$-q$
(0,0)	(1,0)	(2,0)	(4,0)	(8,0)	(16,0)	(32,0)

$$V = \frac{kq}{1} - \frac{kq}{2} + \frac{kq}{4} - \frac{kq}{8} + \frac{kq}{16} \text{----}$$

$$V = kq\left[1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \frac{1}{16} + \text{---}\right] \qquad c.r = -\frac{1}{2}$$

$$a = 1$$

$$V = kq\left[\frac{1}{1-\left(-\frac{1}{2}\right)}\right] \Rightarrow \frac{2kq}{3}$$

Binomial Theorem

$(a+b)^2 = a^2+b^2+2ab$

$(1+x)^2 = 1+x^2+2x$

If $x <<< 1$

$\boxed{(1+x)^2 = 1+2x}$

If n is any number then n^2 must be greater than n → False

$\boxed{n^2 > n}$ → If $n>1$

$\boxed{n^2 = n}$ → If $n=1$

$\boxed{n^2 < n}$ → $0<n<1$

$(1+x)^n = 1+nx$

If $x<<<1$

$(1-x)^n = 1-nx$

$(1-x)^{-n} = 1+nx$

$(1+x)^{-n} = 1-nx$

Que - Binomial Approximation

(i) $(1.002)^3 \rightarrow (1+0.002)^3 \rightarrow 1+3\times0.002$
$\rightarrow 1+0.006$
$\rightarrow 1.006$

(ii) $\sqrt{98} = (100-2)^{1/2} \rightarrow \left[100\left(1-\frac{2}{100}\right)\right]^{1/2} \rightarrow 10\left[1-\frac{2}{100}\right]^{1/2}$

$\rightarrow 10\left[1-\frac{\cancel{2}}{100}\times\frac{1}{\cancel{2}}\right]$

$\rightarrow 10\left[1-\frac{1}{100}\right]$

$\rightarrow 10-\frac{1\cancel{0}}{10\cancel{0}}$

$\rightarrow 10-0.1 \rightarrow 9.9$

Que - acceleration due to gravity at height h given as $g_h = \frac{GM}{(R+h)^2}$ and acceleration due to gravity at earth surface $g_0 = \frac{GM}{R^2}$ then write g_h in term of g_0

if $(h <<< R)$

$$\frac{g_h}{g_0} = \frac{\frac{GM}{(R+h)^2}}{\frac{GM}{R^2}}$$

$$\frac{g_h}{g_0} = \frac{R^2}{(R+h)^2}$$

$$g_h = \frac{g_0 R^2}{(R+h)^2} = \frac{g_0 R^2}{R^2\left(1+\frac{h}{R}\right)^2}$$

$$g_h = g_0\left(1+\frac{h}{R}\right)^{-2}$$

$$g_h = g_0\left(1-\frac{2h}{R}\right)$$

Que- $y = \frac{K}{(R+d)^2}$, find y if $d << R$ and K is constant write & calculate

$$y = \frac{K}{(R+d)^2}$$

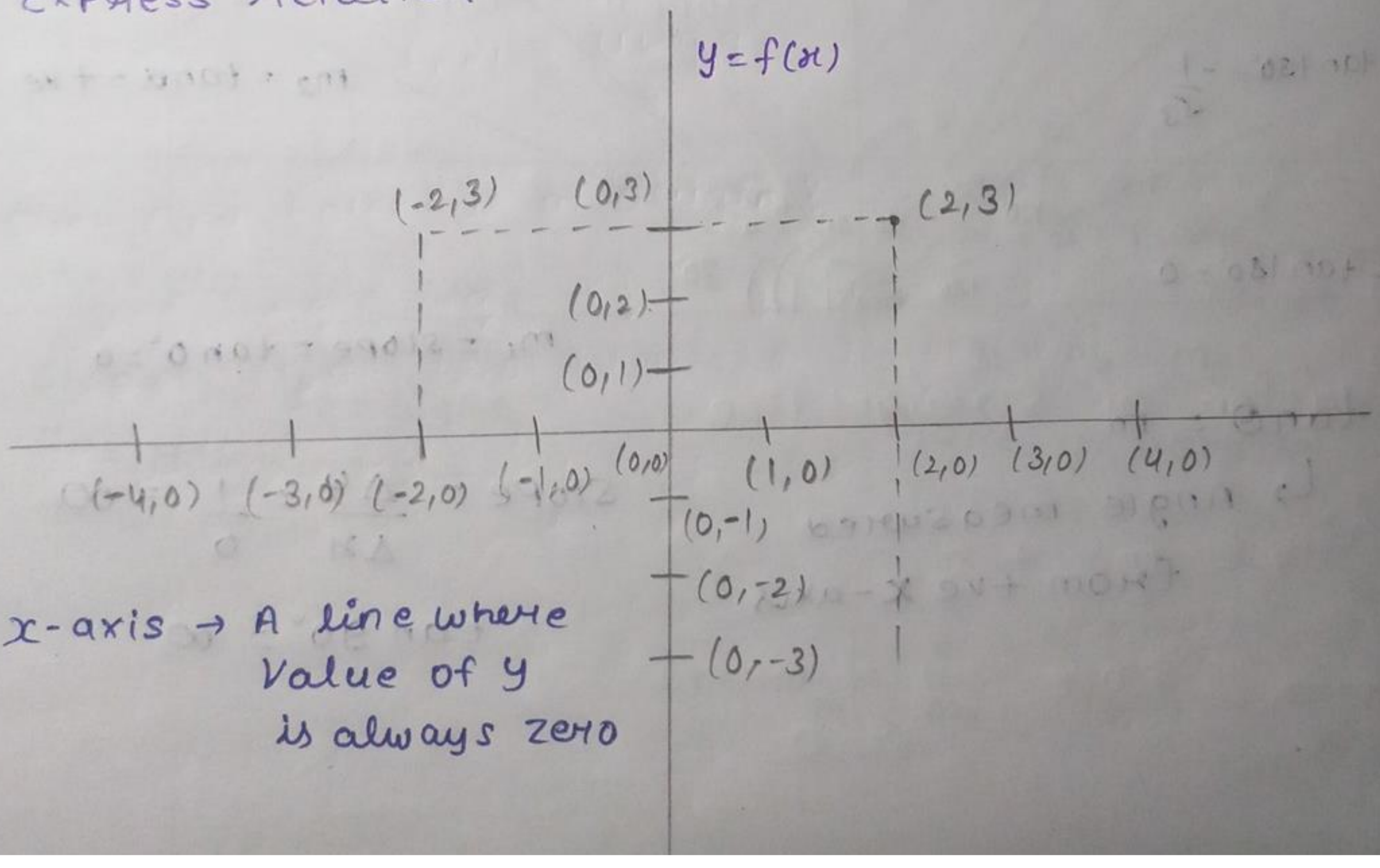
$$y = \frac{K}{R^2\left(1+\frac{d}{R}\right)^2}$$

$$= \frac{K}{R^2}\left(1+\frac{d}{R}\right)^{-2}$$

$$y = \frac{K}{R^2}\left[1-\frac{2d}{R}\right]$$

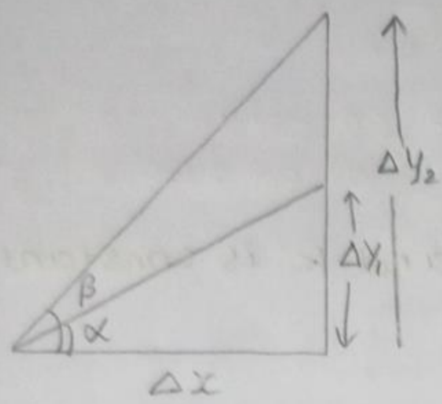
Co-ordinate Geometry

Express relation between two physical quantity



x-axis → A line where Value of y is always zero

	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$\frac{-1}{\sqrt{3}}$	0



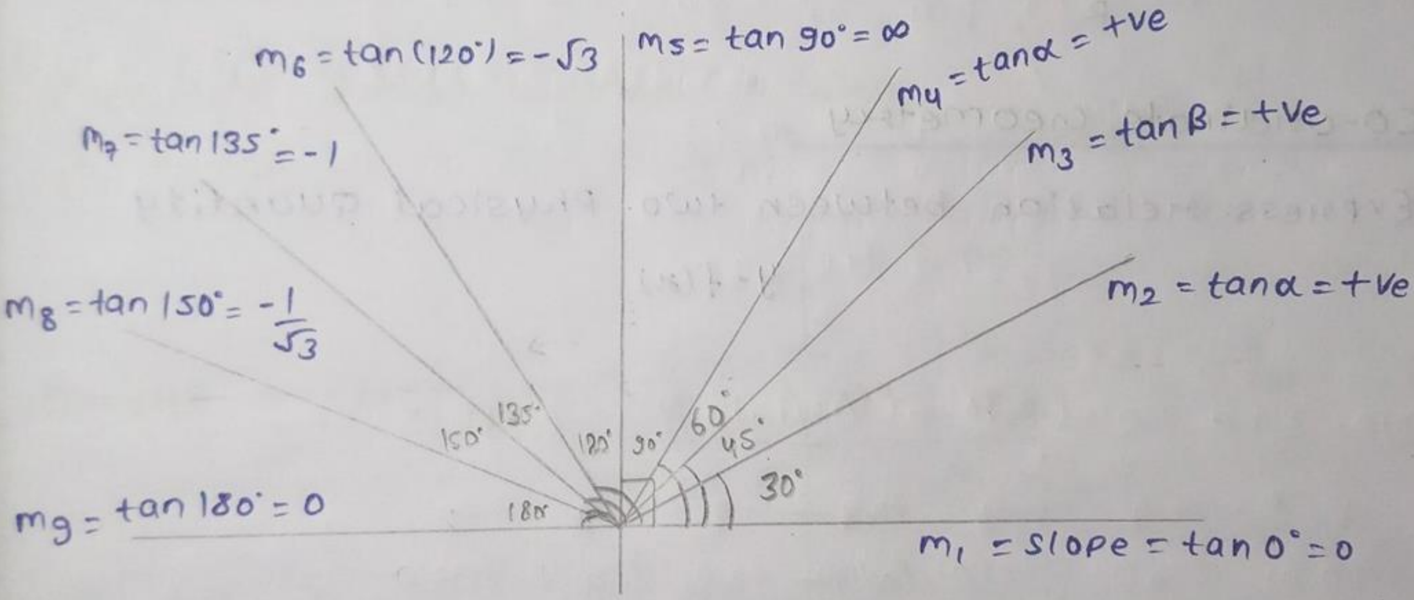
$\theta \uparrow$ ($0°$ to $90°$)

$\tan\theta \uparrow$ (m)

$$\text{slope} = \frac{\Delta y}{\Delta x} = \tan\theta$$

Slope $\propto \Delta y$

$\tan\alpha = \frac{\Delta y_1}{\Delta x}$ $\qquad$ $\tan\beta = \frac{\Delta y_2}{\Delta x}$



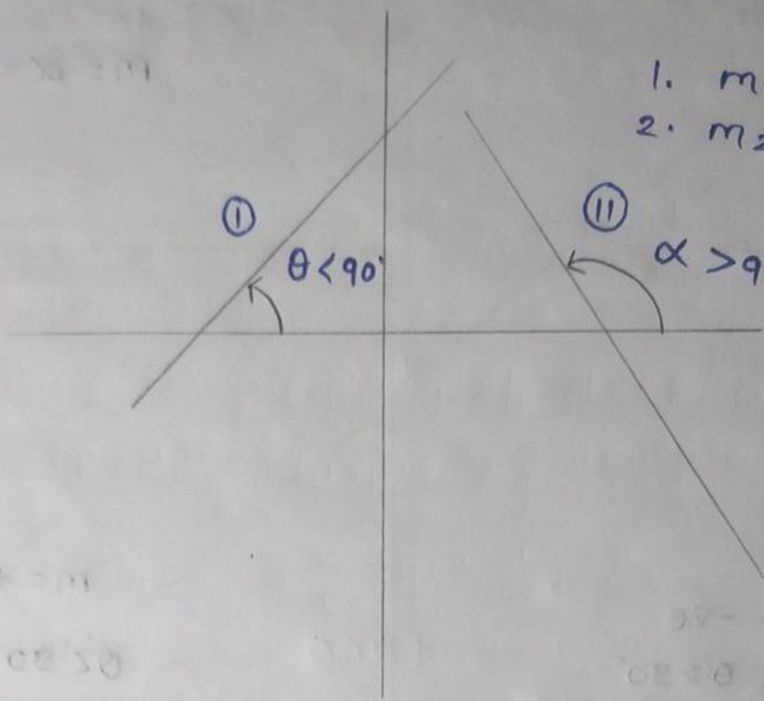
$\tan\theta = m$

↳ Angle measured from +ve x-axis

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{1}{0} = \infty$$

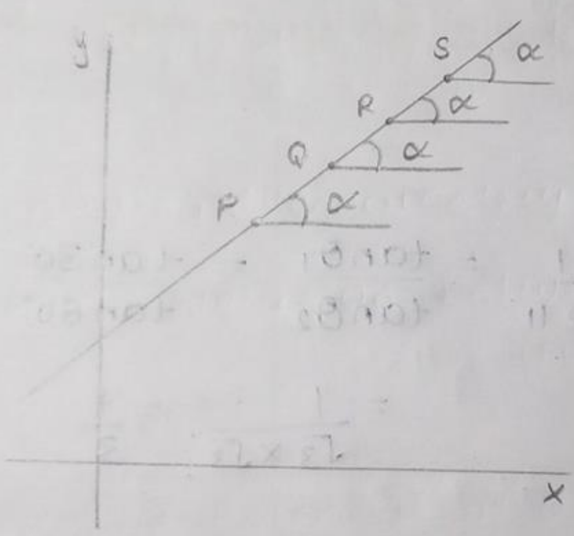
$$\tan 90° = \infty$$

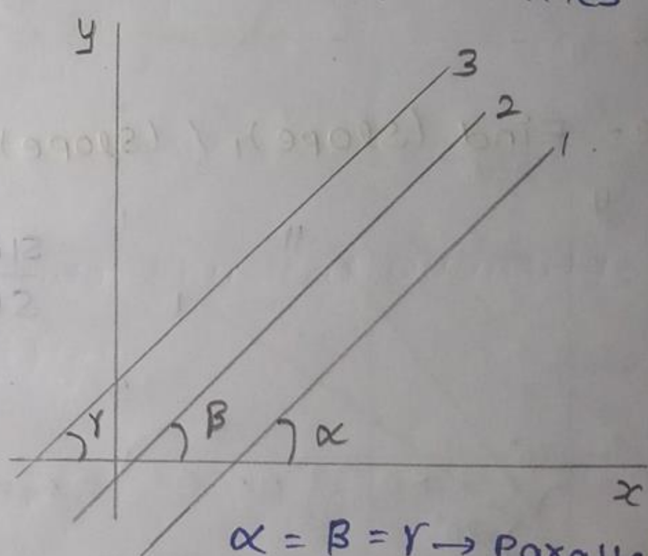
Graph of straight line (linear relation between Physical Quantity)



1. $m_1 = \tan\theta = +ve$
2. $m_2 = \tan\alpha = -ve$

Parallel lines





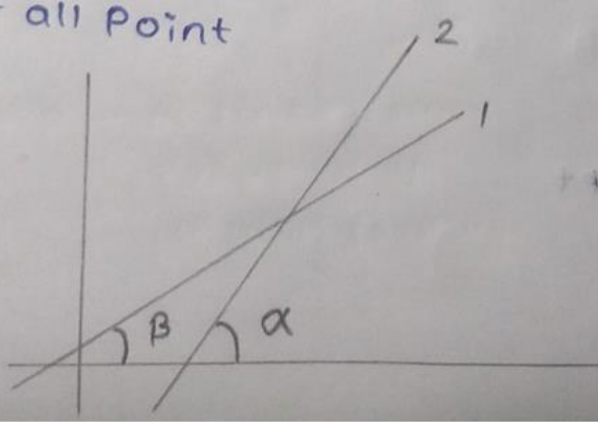
$\alpha = \beta = \gamma \rightarrow$ Parallel lines

$m_P = m_Q = m_R = m_S$

$m_1 = m_2 = m_3$

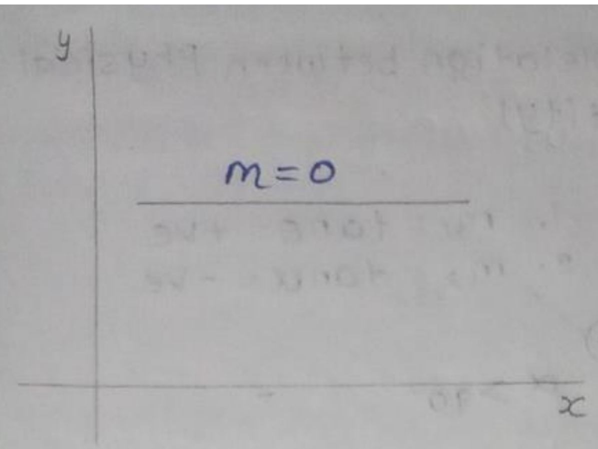
Slope of straight line remains same at all point

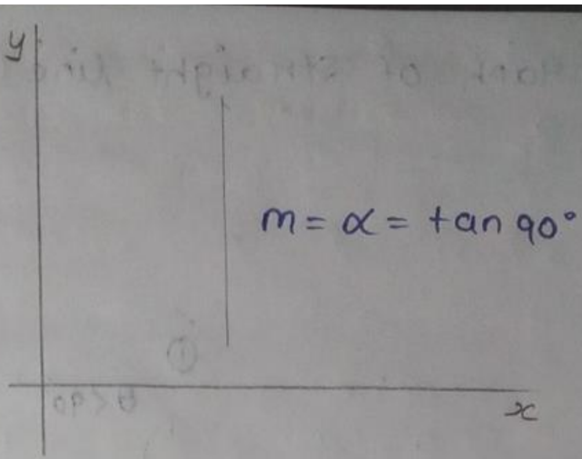
Slope of parallel lines remain same

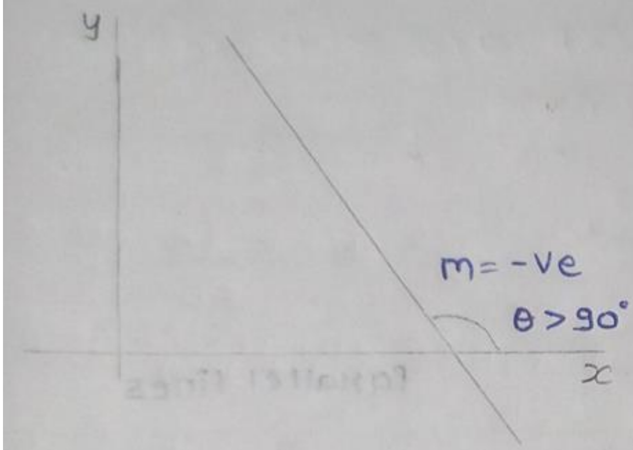


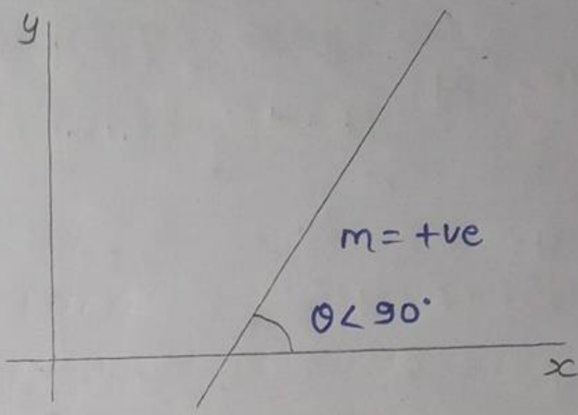
$\beta < \alpha$

$m_1 < m_2$

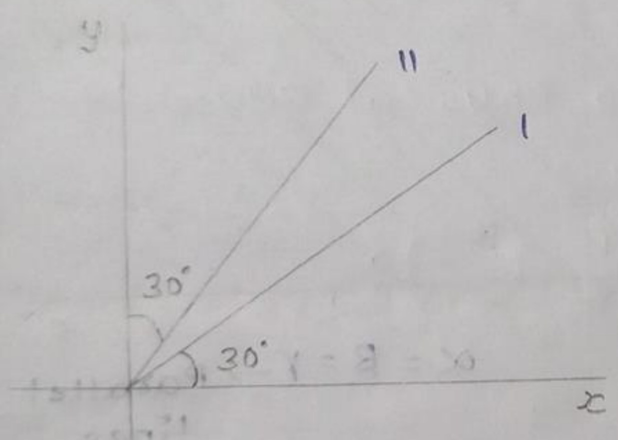








Que- Find $(\text{slope})_{I} / (\text{slope})_{II}$ =



$$\frac{\text{slope I}}{\text{slope II}} = \frac{\tan\theta_1}{\tan\theta_2} = \frac{\tan 30^\circ}{\tan 60^\circ}$$

$$= \frac{1}{\sqrt{3}\times\sqrt{3}} = \frac{1}{3}$$

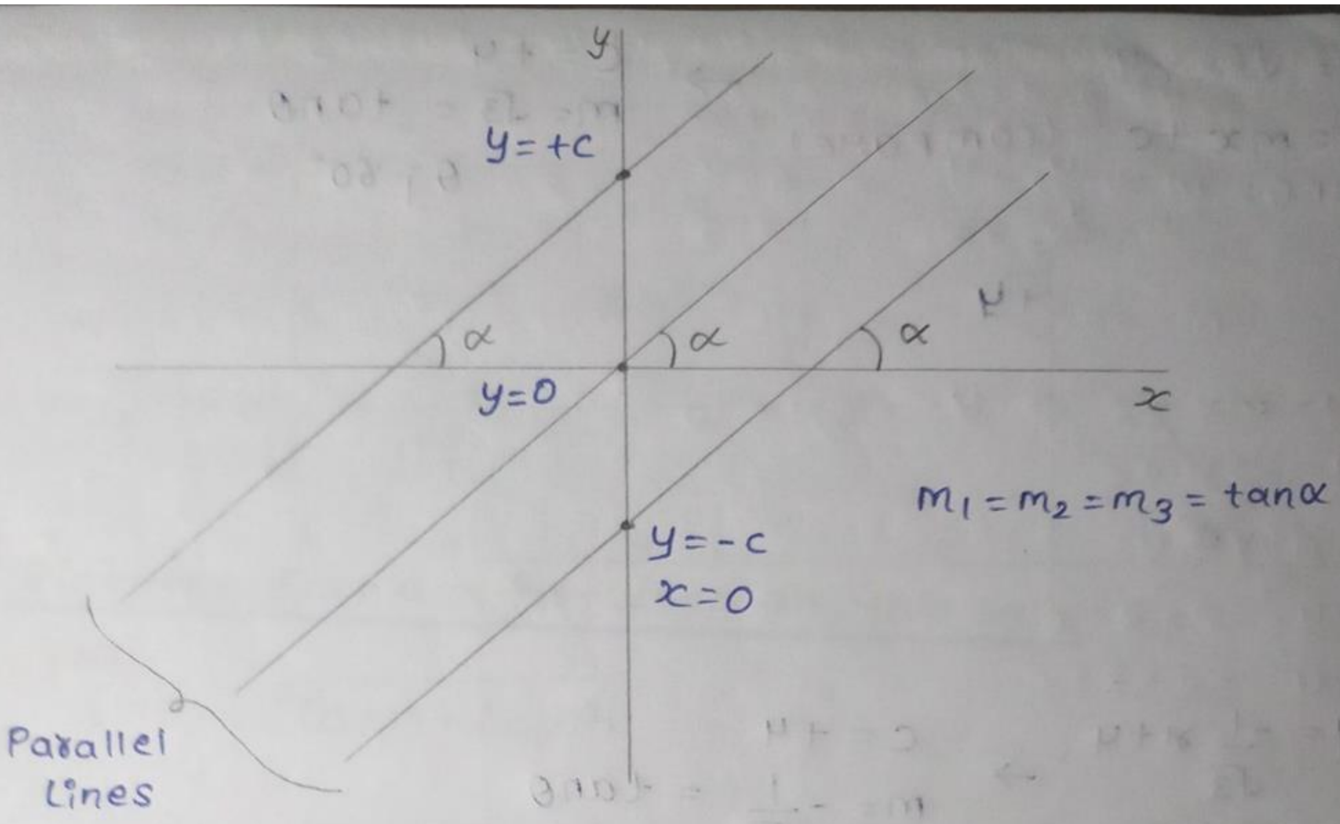
Equation of Straight line

$$y = mx + c$$

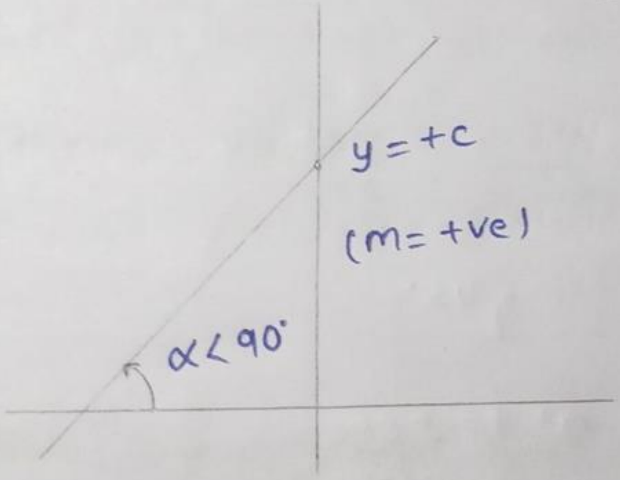
If $x = 0$
$y = c$

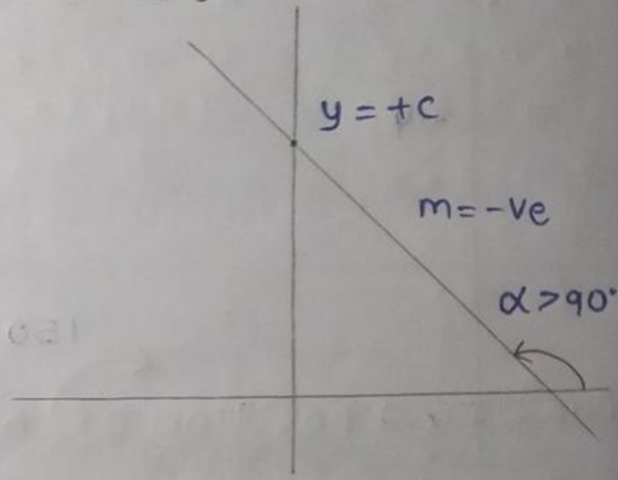
m ↑ slope

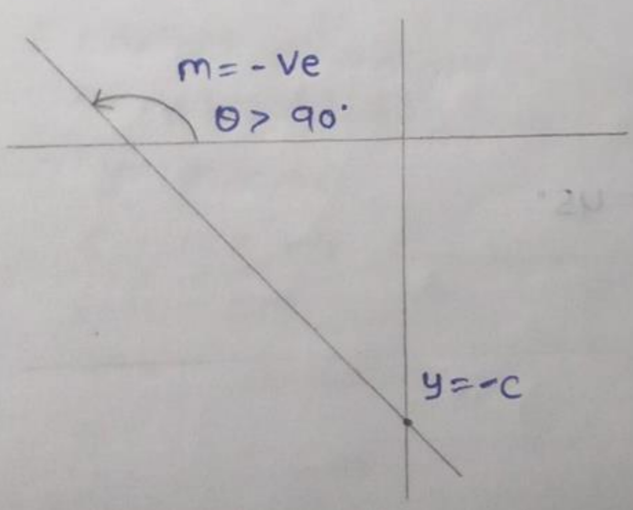
c ↑ y-Intercept
$x = 0$

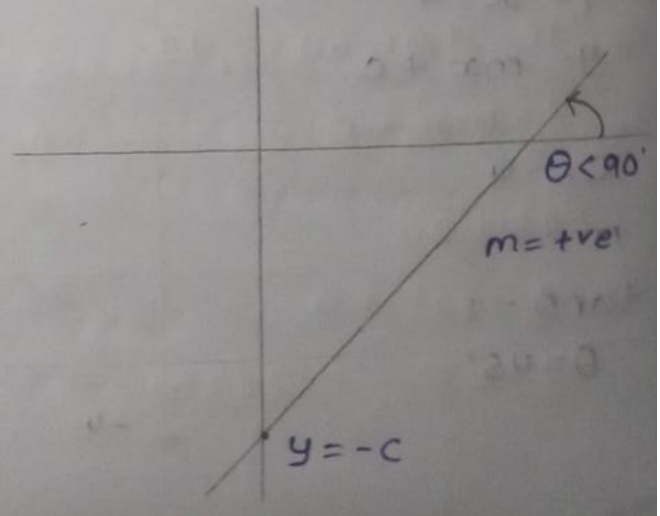


Que- comment on slope and intercept







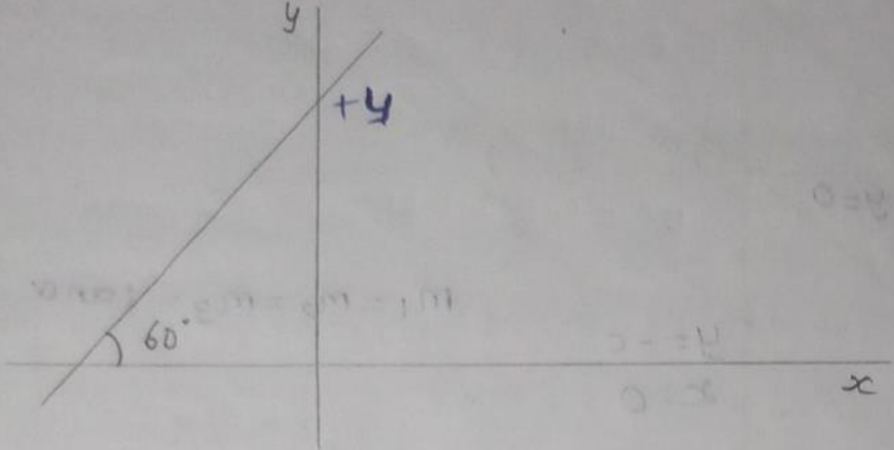


$y = \sqrt{3}x + 4$

$y = mx + c$ (compare) $\rightarrow$ $c = +4$

$m = \sqrt{3} = \tan\theta$

$\theta = 60^\circ$

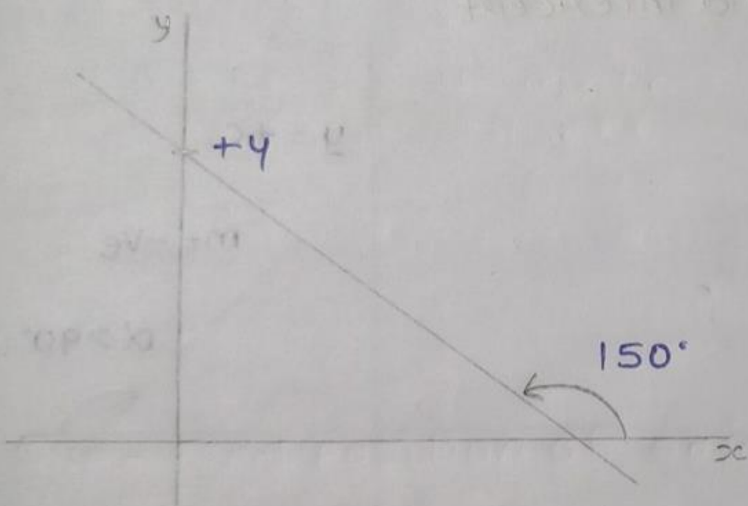


$y = -\frac{1}{\sqrt{3}}x + 4$

$y = mx + c$ $\rightarrow$ $c = +4$

$m = -\frac{1}{\sqrt{3}} = \tan\theta$

$\theta = 150^\circ$



$y = x - 4$

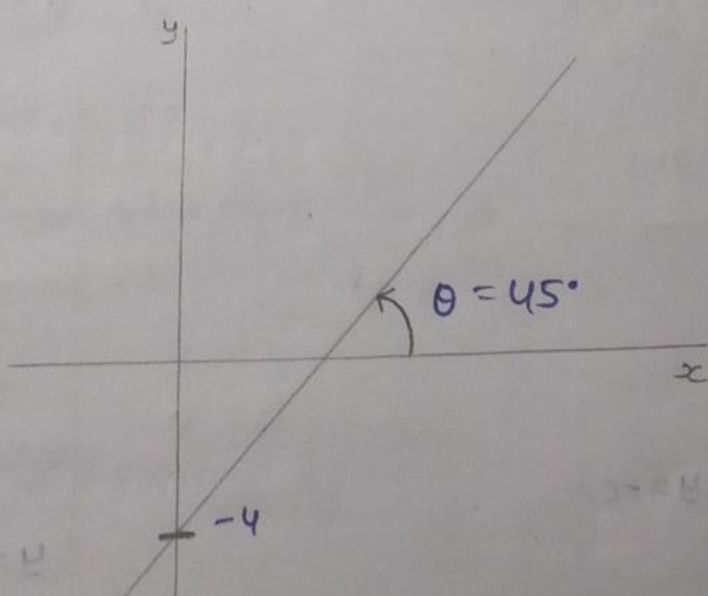
$y = mx + c$

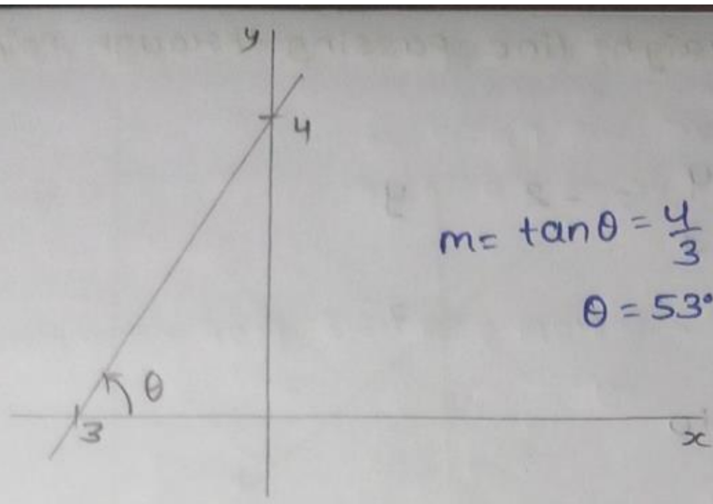
$c = -4$

$m = 1$

$\tan\theta = 1$

$\theta = 45^\circ$





$$m = \tan\theta = \frac{4}{3}$$
$$\theta = 53^\circ$$

$$y = mx + c$$
$$m = \frac{4}{3}, \quad c = 4$$
$$y = \frac{4}{3}x + 4$$

Distance Formula between any two point

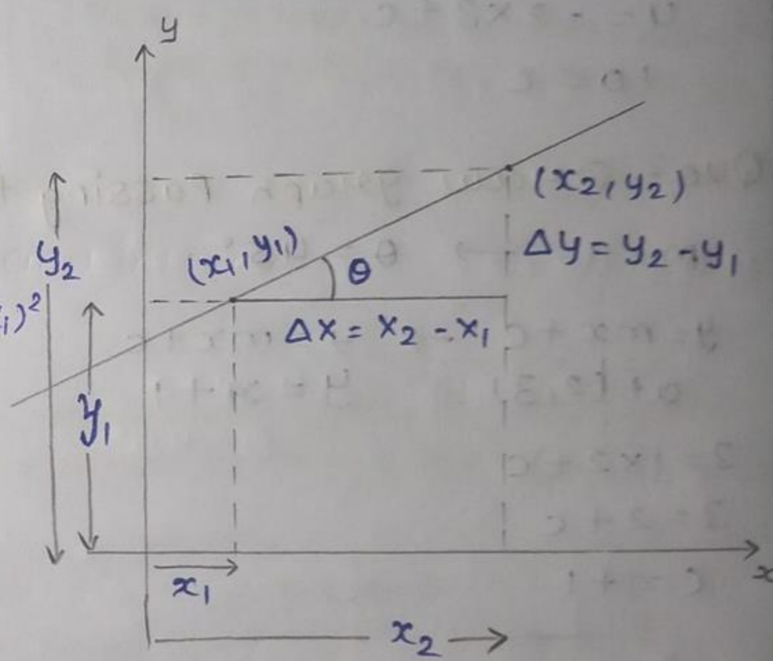
MR Ratta

$$\text{Dist}^n = \sqrt{(\Delta x)^2 + (\Delta y)^2}$$

* $(x_1, y_1, z_1) \longrightarrow (x_2, y_2, z_2)$

$$\text{dist}^n = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$m = \tan\theta = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



Que- Draw graph having Y - intercept 4 and Passing through (2, 6)

$c = 4$

$(x, y) = (2, 6)$

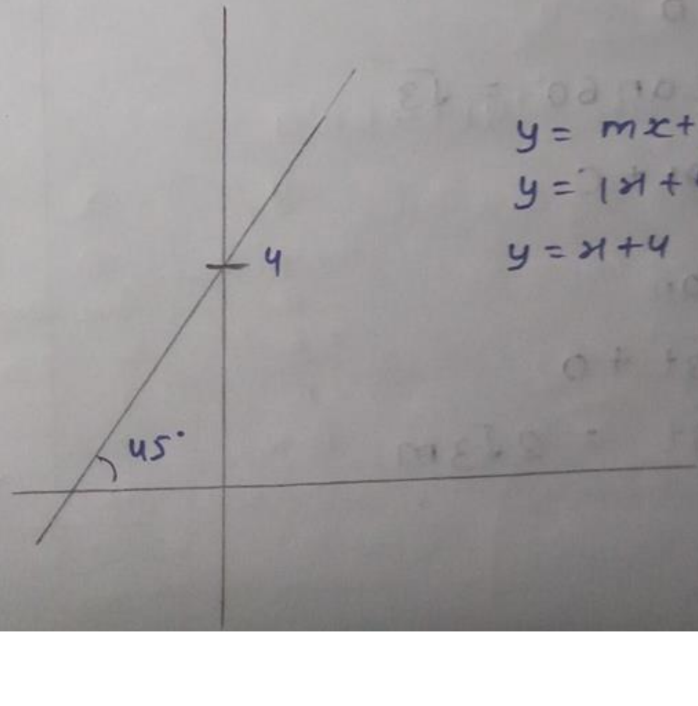
$y = mx + c$

$6 = m2 + 4$

$6 - 4 = 2m$

$m = 1$

$\theta = 45^\circ$



$y = mx + c$

$y = 1x + 4$

$y = x + 4$

Que- Find equation of a straight line passing through point (3, 4) and (2, 6)

$$\tan\theta = m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-4}{2-3} = -2$$

$m = -2$

$y = mx + c \rightarrow y = -2x + 10$

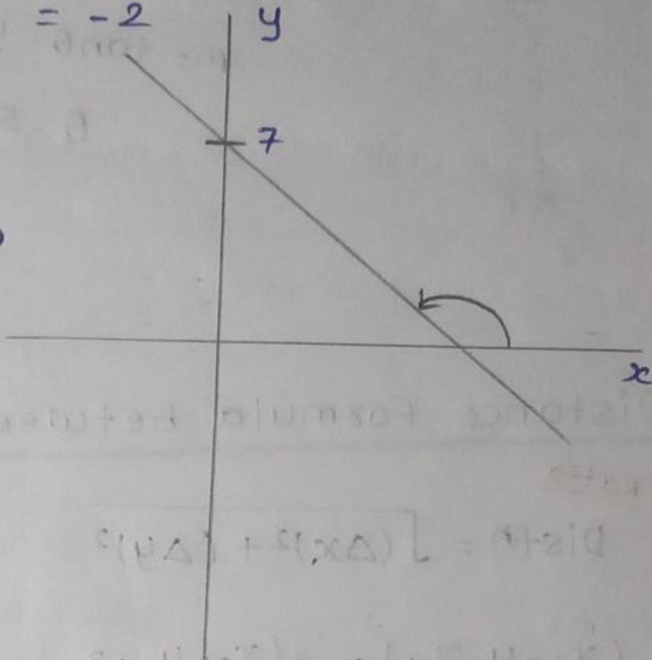
$y = -2x + c$

at $(3, 4)$

$y = -2 \times 3 + c$

$4 = -2 \times 3 + c$

$10 = c$



Que- Draw graph passing through (2,3) and slope 1

$m = 1 \longrightarrow \theta = 45°$

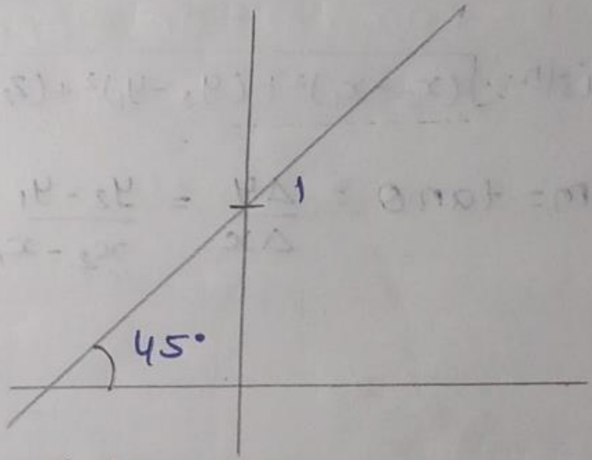
$y = mx + c \rightarrow y = mx + c$

at $(2, 3)$ $\quad y = x + 1$

$3 = 1 \times 2 + c$

$3 = 2 + c$

$c = +1$



Que- Find position of object at $t = 2$ sec

$c = 0$

$m = \tan 60° = \sqrt{3}$

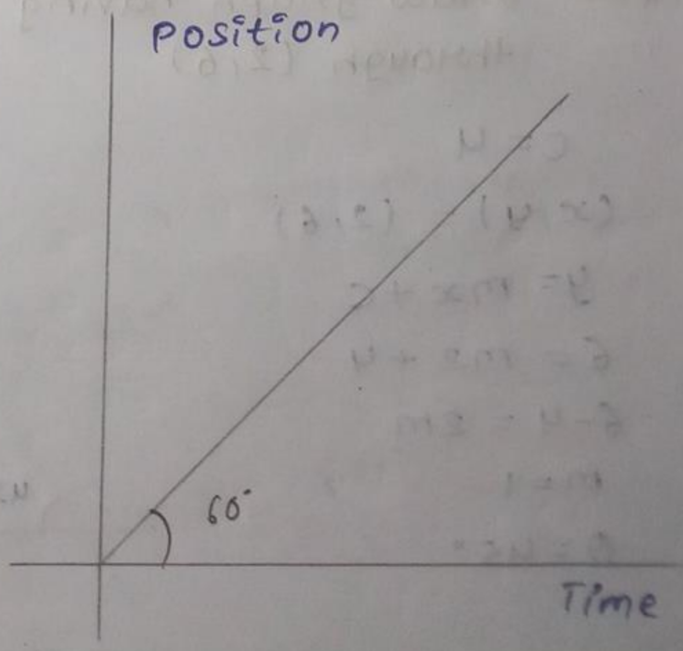
$y = mx + c$

$\downarrow$

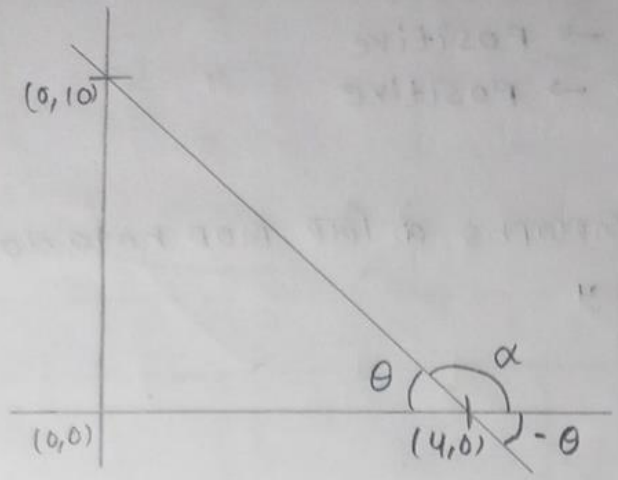
Position

$y = \sqrt{3}t + 0$

$y = \sqrt{3}t = 2\sqrt{3}\,m$



Que- Find slope of graph and value of y at $x=2$



$m = -\frac{5}{2}, \ c = 10$

$y = mx + c \rightarrow y = -\frac{5}{2}x + 10$

$y_{\text{at } x=2} \rightarrow = -\frac{5}{2} \times 2 + 10$

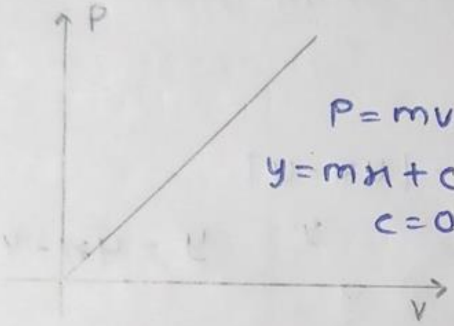
$= 5$

$\tan(-\theta) = -\tan\theta$

$m = -\tan\theta$

$m = -\frac{5}{2}$

Que - Draw graph b/w momentum & velocity

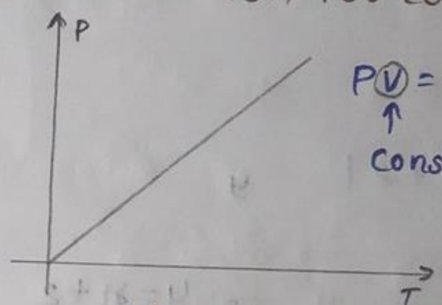


$P = mv$

$y = mx + c$

$c = 0$

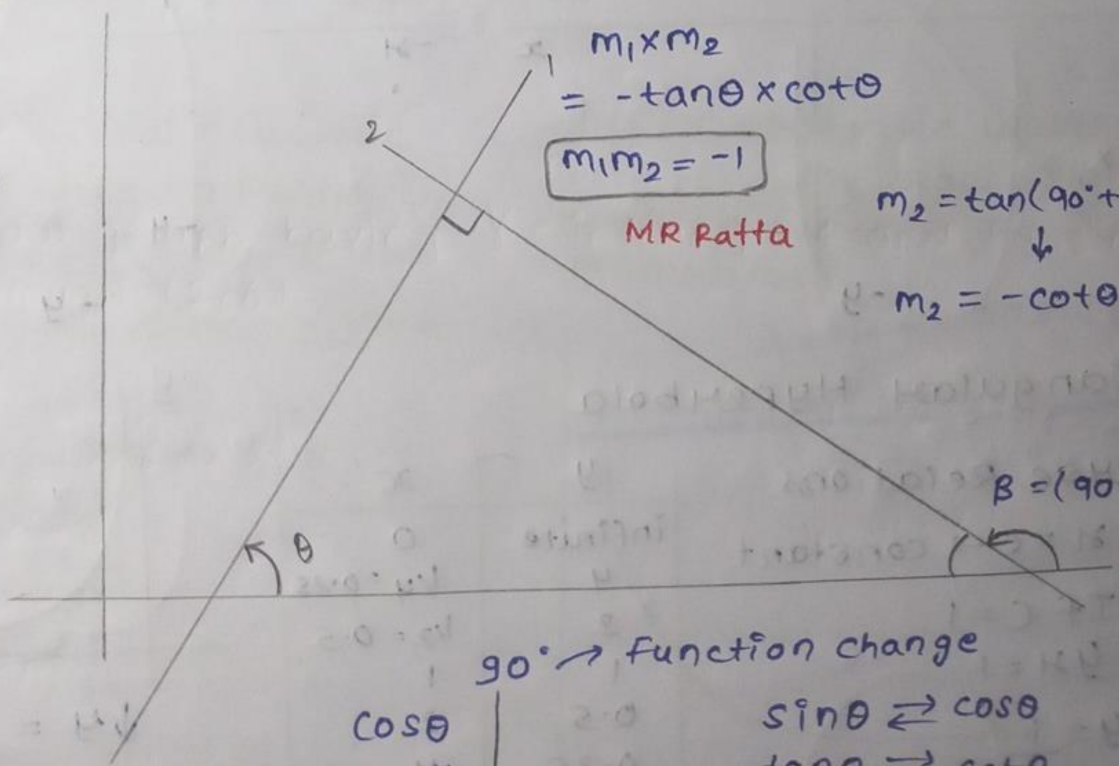
Draw graph b/w Pressure & temptation for constant vol.



$PV = nRT$

constant

Que- If two straight line perpendicular to each other then prove that product of their slope is -1



$m_1 \times m_2$

$= -\tan\theta \times \cot\theta$

$\boxed{m_1 m_2 = -1}$

MR Ratta

$m_2 = \tan(90° + \theta)$

$\downarrow$

$m_2 = -\cot\theta$

$90° \rightarrow$ Function change

cosθ

-ve

tanθ

$\sin\theta \rightleftarrows \cos\theta$

$\tan\theta \rightleftarrows \cot\theta$

Graph of |x|

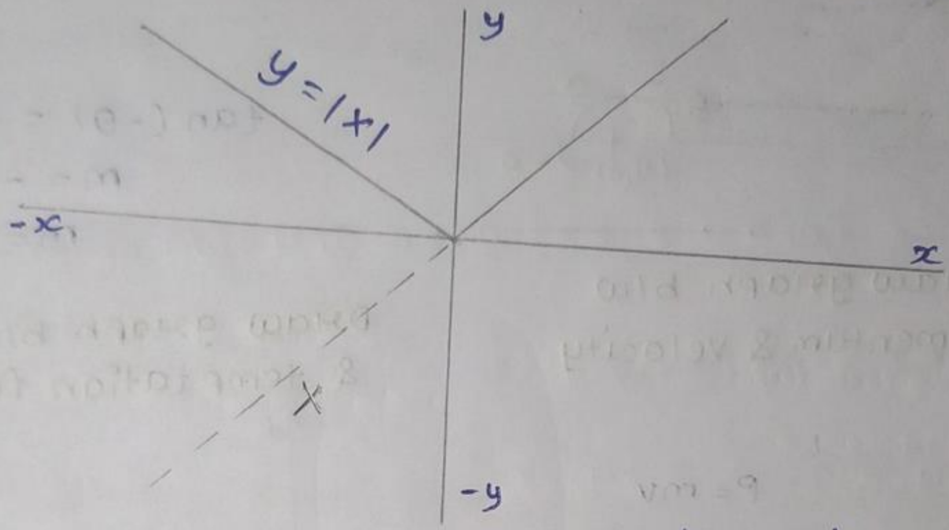
$y = |x| \rightarrow$ mod $\rightarrow$ Positive $\rightarrow$ Positive

Negative $\rightarrow$ Positive

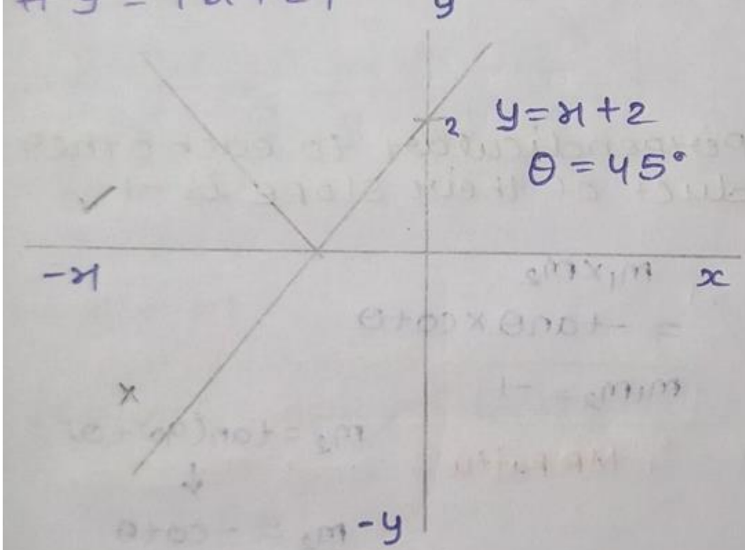
$y = |+1| = 1$

$y = |-1| = 1$

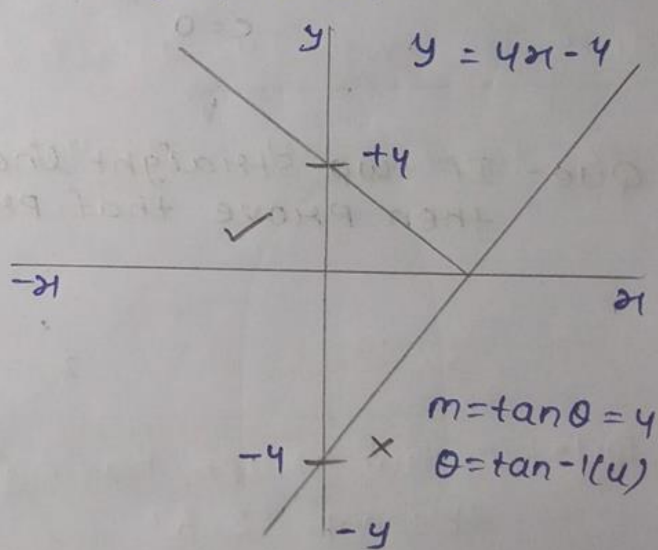
MR*- Graphs के लिए MOD hata do



$y = |x+2|$



$y = |4x-4|$



Rectangular Hyperbola

Inverse Relations

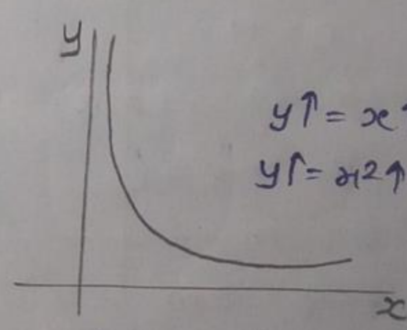
$yx = c \rightarrow$ constant

If $C = 1$

$yx = 1$

$y = \frac{1}{x}$

y	x
infinite	0
4	1/4 = 0.25
2	1/2 = 0.5
1	1
0.5	2
0.25	4
0.1	10
0	∞

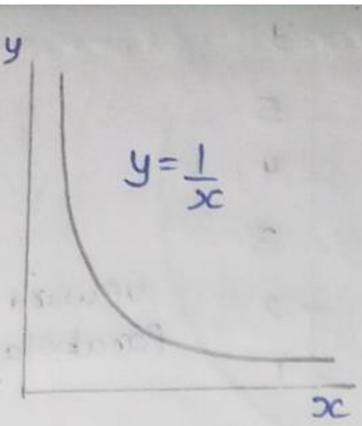


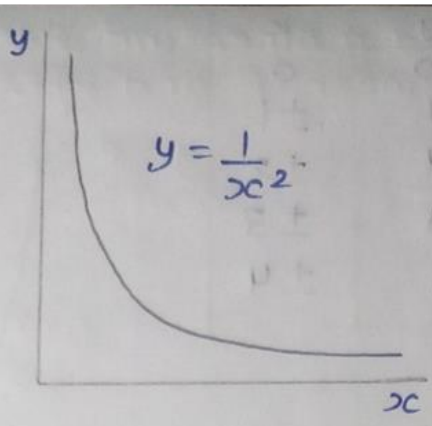
$y\uparrow = x\uparrow$

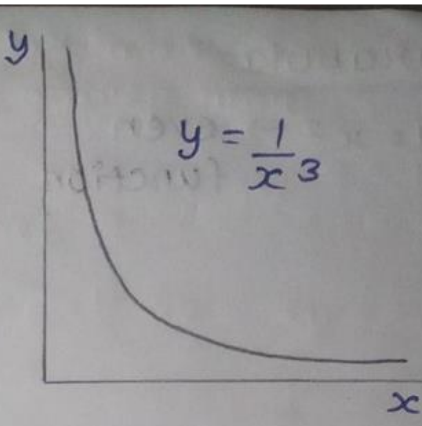
$y\uparrow = x^2\uparrow$

$\downarrow y = \frac{1}{x\uparrow}$

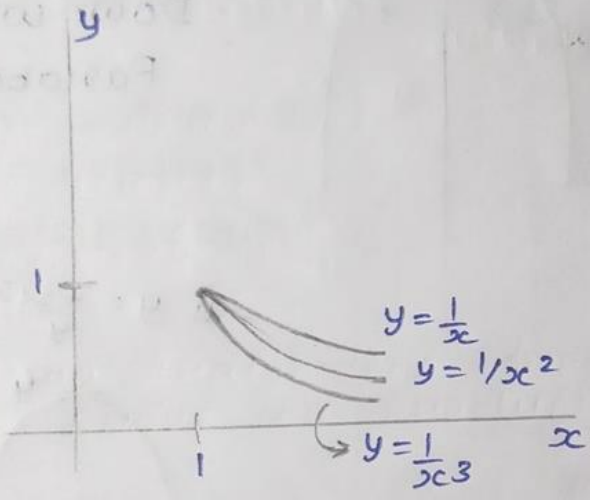
$\uparrow y = \frac{1}{x\downarrow}$





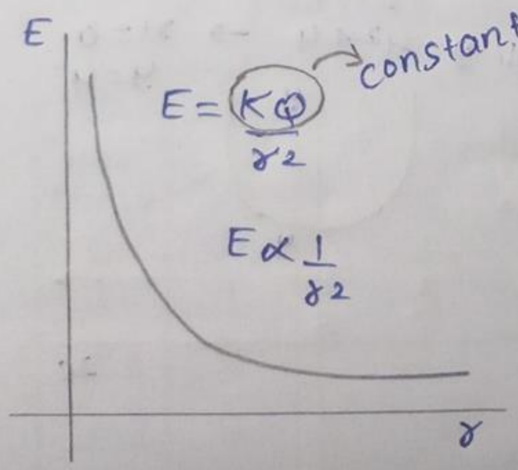


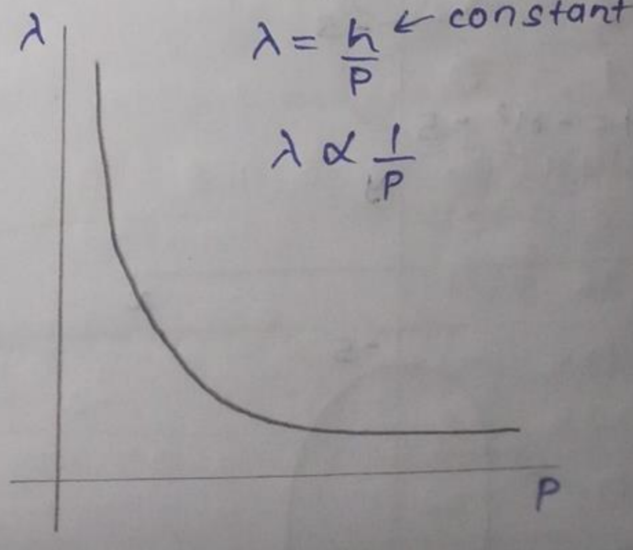
$y=\frac{1}{x}$	$y=\frac{1}{x^2}$	$y=\frac{1}{x^3}$
$x=10$ $y=0.1$	$x=10$ $y=0.01$	$x=10$ $y=0.001$



Que- we know electric field due to point charge $E=\frac{KQ}{r^2}$ then, draw electric field distance graph

Que- De-broglie wavelength $\lambda=h/p$. Draw graph between wavelength and momentum.

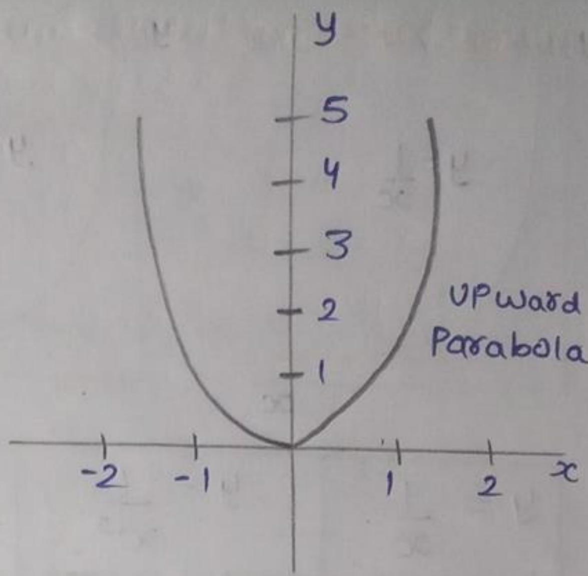




Parabola

\# $y = x^2 \rightarrow$ even function

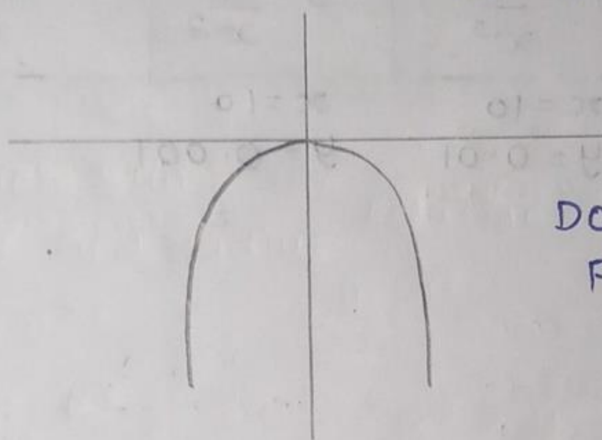
y	x
0	0
1	±1
4	±2
9	±3
16	±4



Upward Parabola

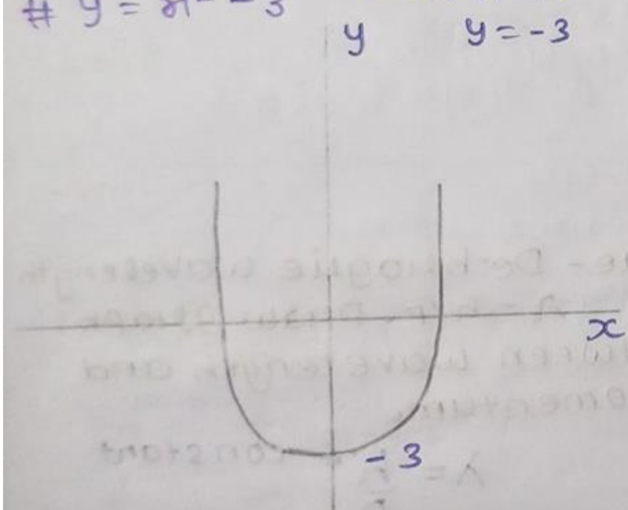
\# $y = -x^2$

y	x
0	0
-1	∓1
-4	∓2
-9	±3

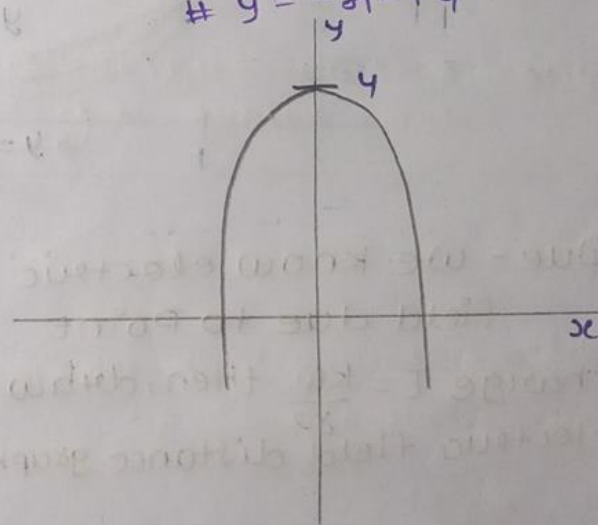


Downward Parabola

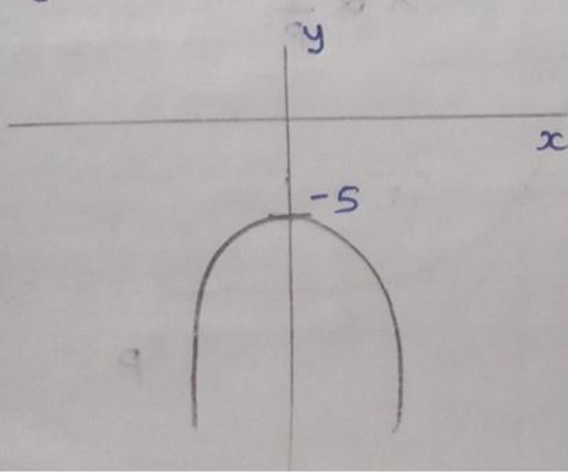
\# $y = x^2 - 3 \rightarrow$ at $x = 0$, $y = -3$



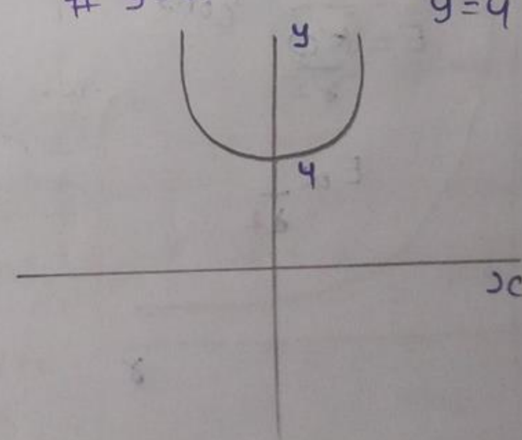
\# $y = -x^2 + 4$



\# $y = -x^2 - 5$

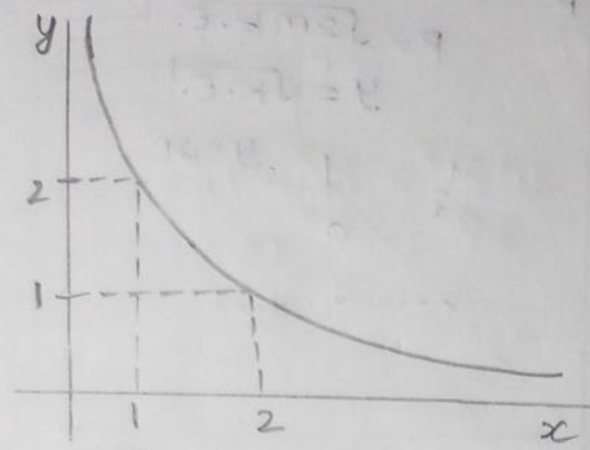


\# $y = x^2 + 4 \rightarrow x = 0$, $y = 4$

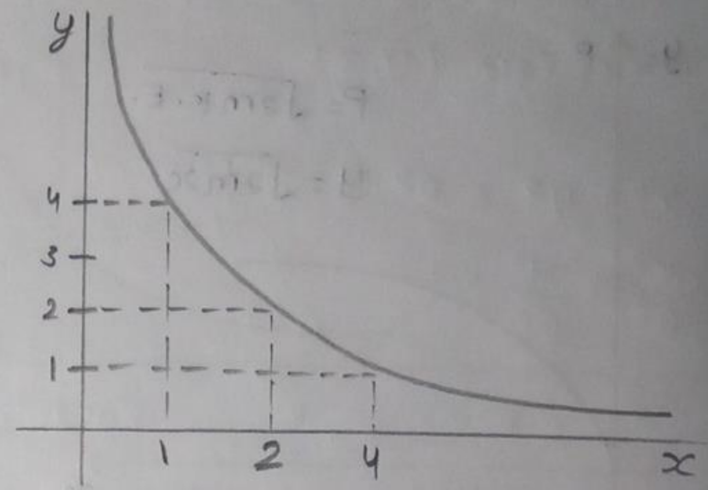


Draw graph

(i) $yx = 2$



(ii) $yx = 4$



Que- Draw graph b/w pressure and volume for Isothermal Process

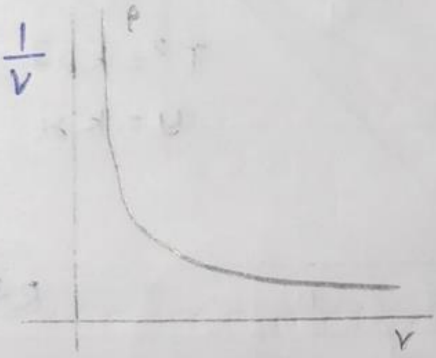
$PV = nRT$

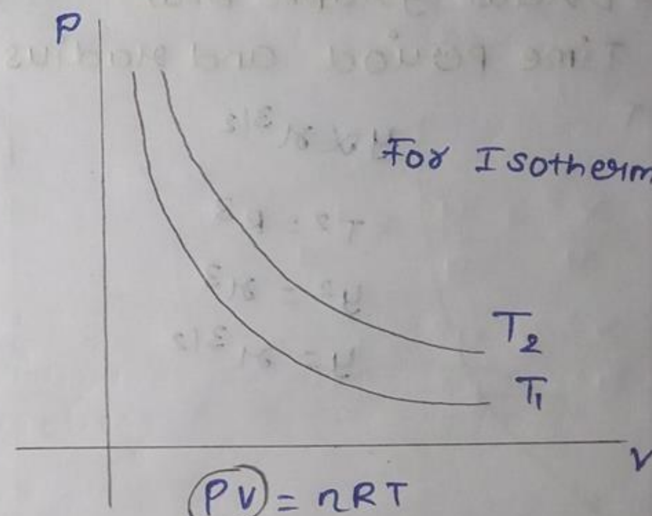
Isothermal process

$T =$ constant

$PV = (\text{constant}) = nRT$

$P \propto \frac{1}{V}$





For Isothermal

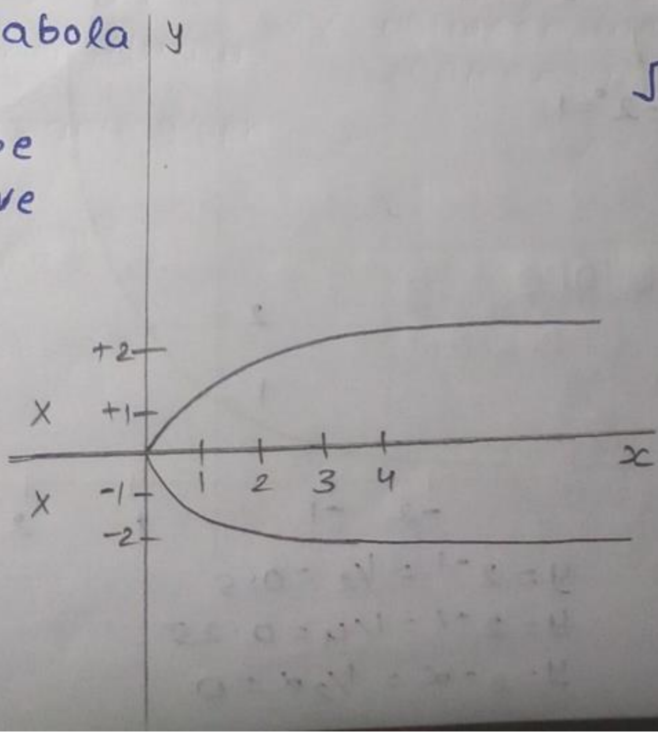
$\text{PV} = nRT$

$T_2 > T_1$

$y = \sqrt{x} \rightarrow$ parabola

$y^2 = x$ ↳ Must be positive

y	x
Not possible	-4
0	0
± 1	1
± 2	4
± 3	9



$\sqrt{-x} \rightarrow$ may possible

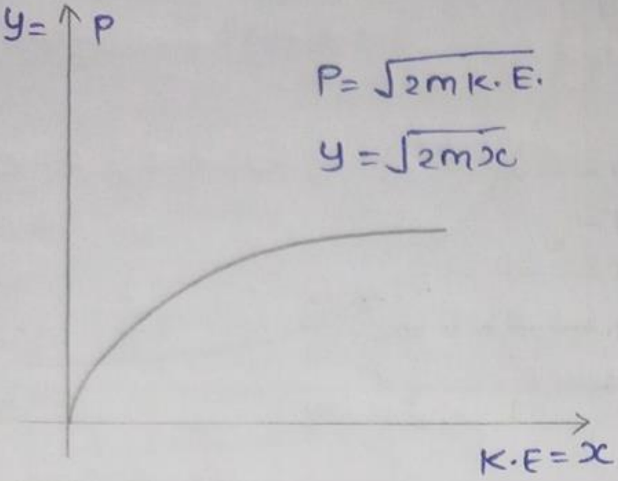
If $x = -ve$

If $x = -4$

$\Rightarrow \sqrt{-(-4)}$

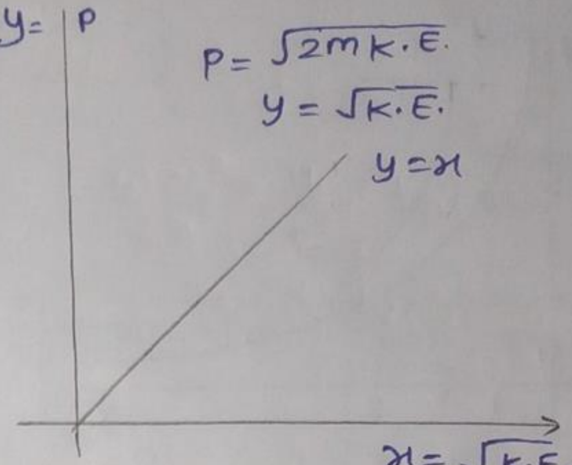
$\Rightarrow \sqrt{4} = 2$

Que- we know K.E. and momentum relation $P=\sqrt{2mK.E.}$
Draw graph between P and K.E.



$P=\sqrt{2mK.E.}$

$y=\sqrt{2mx}$

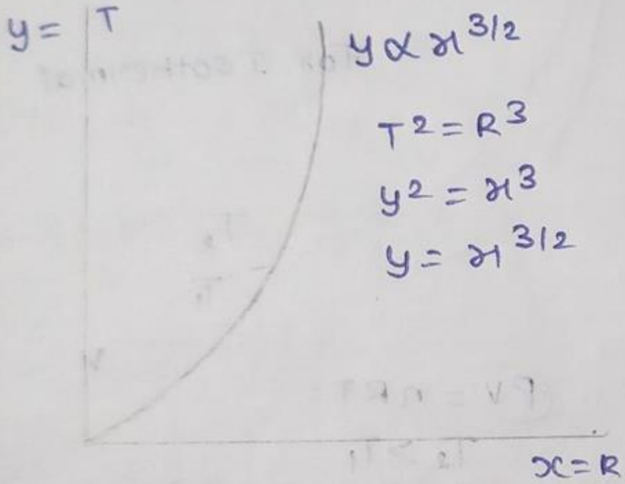


$P=\sqrt{2mK.E.}$

$y=\sqrt{K.E.}$

Que- Draw graph b/w Time period and radius

Relation ⇒ $T^2 = kR^3$ (k: constant)

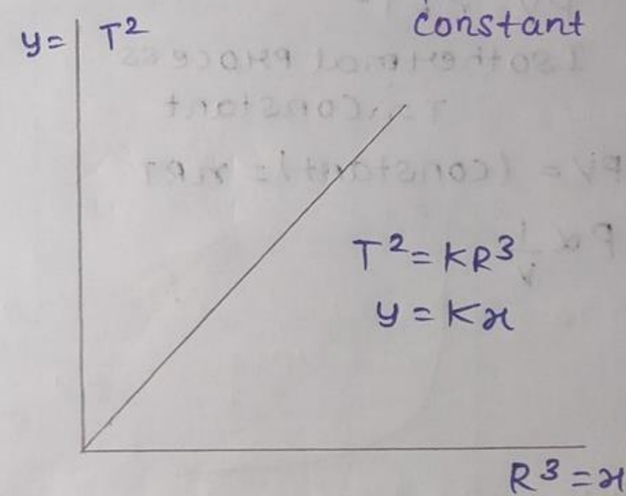


$y \propto x^{3/2}$

$T^2 = R^3$

$y^2 = x^3$

$y = x^{3/2}$



$T^2 = kR^3$

$y = kx$

$\underline{y = 2^x}$ $\quad y = e^x = (2.71)^x$

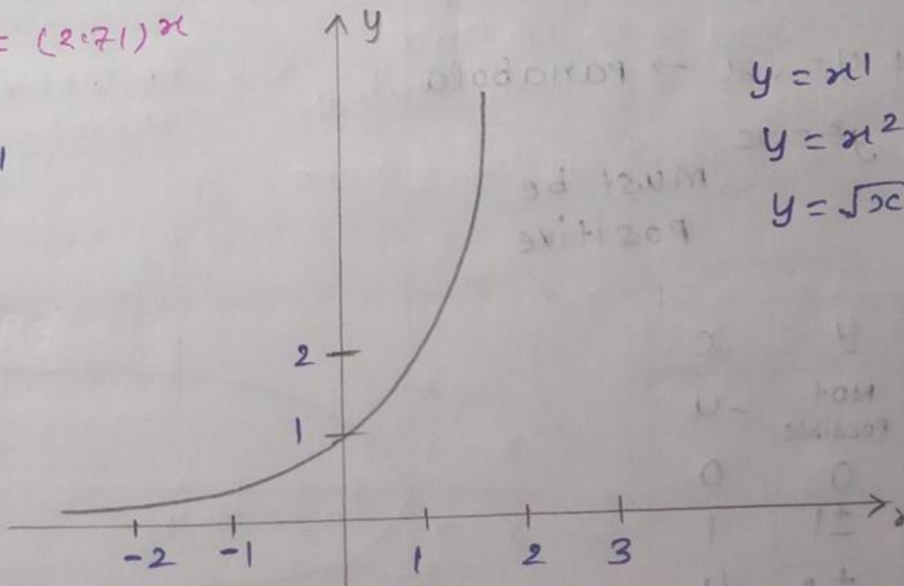
y	x
1	0
2	1
8	3
16	4
0.5	-1
0.25	-2
0	$-\infty$

$y = 2^0 = 1$

$y = x^1$

$y = x^2$

$y = \sqrt{x}$



$y = 2^{-1} = 1/2 = 0.5$

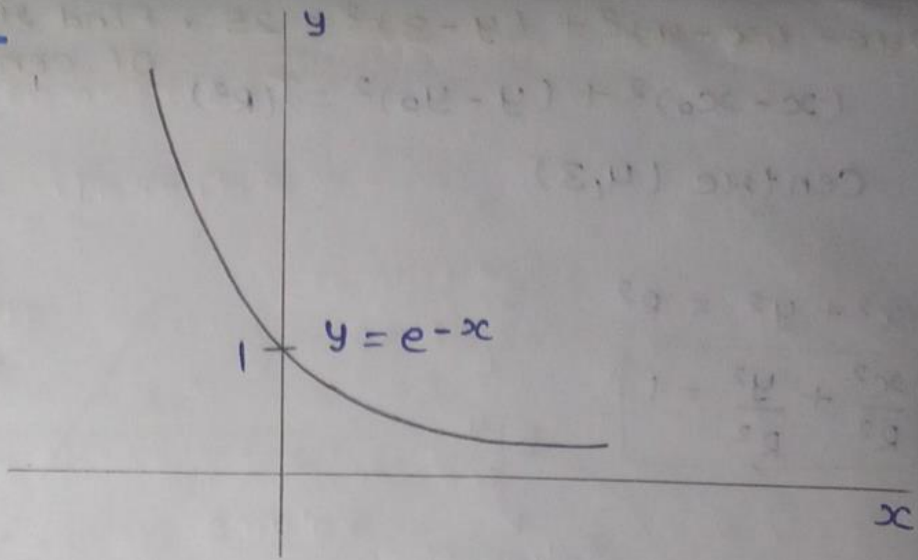
$y = 2^{-2} = 1/4 = 0.25$

$y = 2^{-\alpha} = 1/2^{\alpha} = 0$

Exponential Function

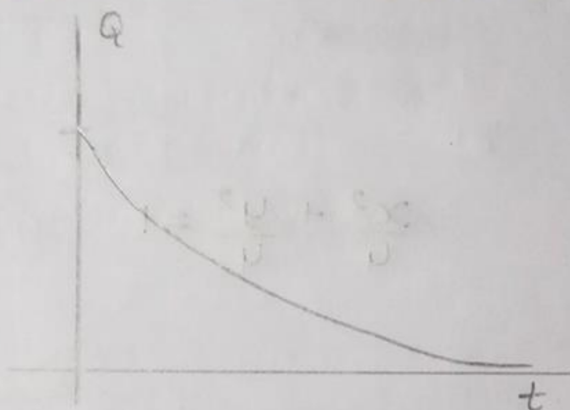
$y = e^{-x}$

y	x
	0
	-1
	+1
	$+\infty$
	$-\infty$

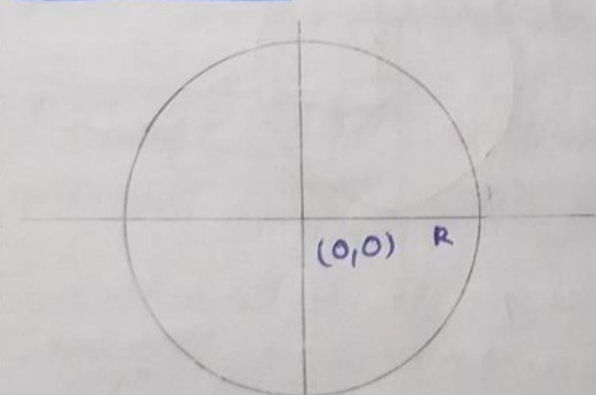


Que- Draw graph b/w Q and t → $Q = Q_0 e^{-t/RC}$

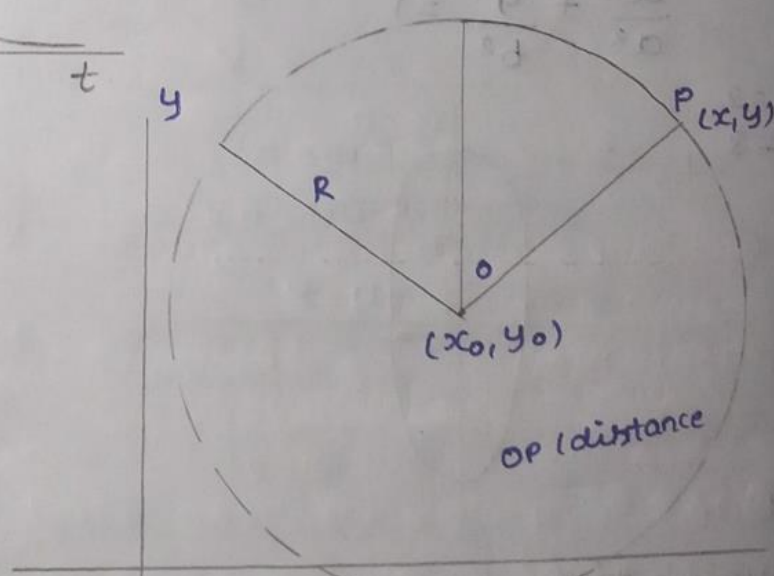
$Q \propto e^{-t}$ → RC = constant



Circular



$x^2 + y^2 = R^2$



$OP = dist^n = R = \sqrt{(x-x_0)^2 + (y-y_0)^2}$

$R^2 = (x-x_0)^2 + (y-y_0)^2$

If $(x_0, y_0) = (0,0)$ → origin

Equation of circle $\boxed{R^2 = x^2 + y^2}$

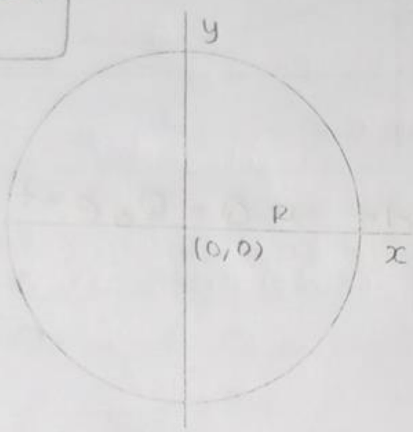
Que- $(x-4)^2 + (y-3)^2 = 25$. Find radius and co-ordinate of centre of circle.

$(x-x_0)^2 + (y-y_0)^2 = (R^2)$

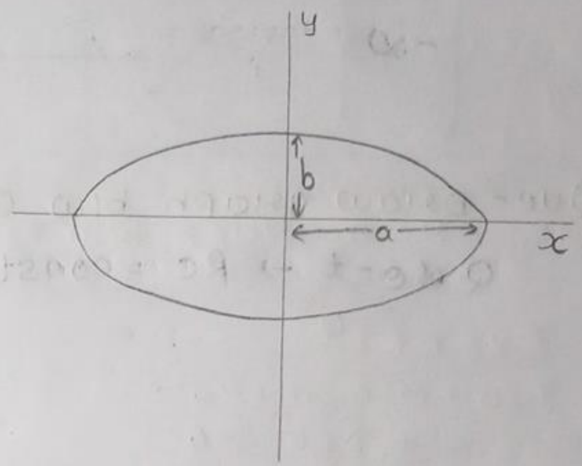
Centre $(4,3)$

$x^2 + y^2 = R^2$

$$\boxed{\frac{x^2}{R^2} + \frac{y^2}{R^2} = 1}$$



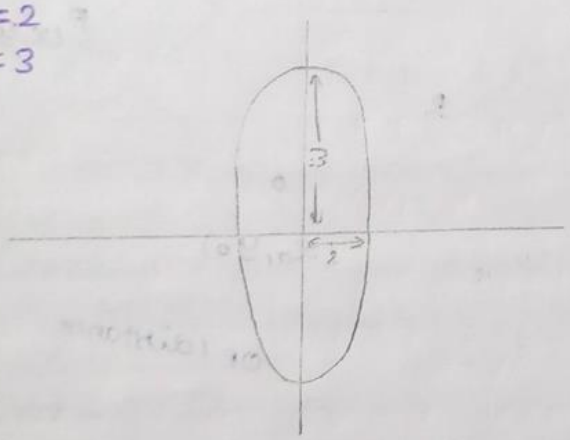
$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$



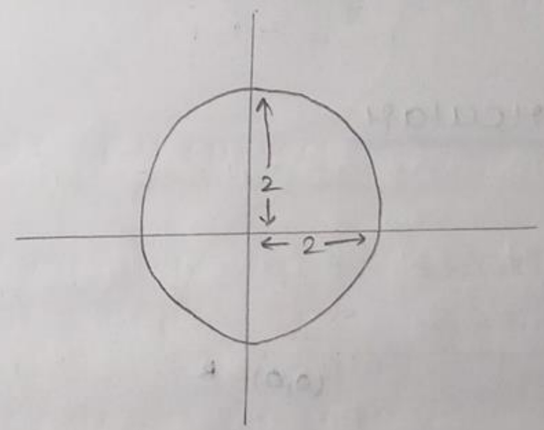
Que- $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

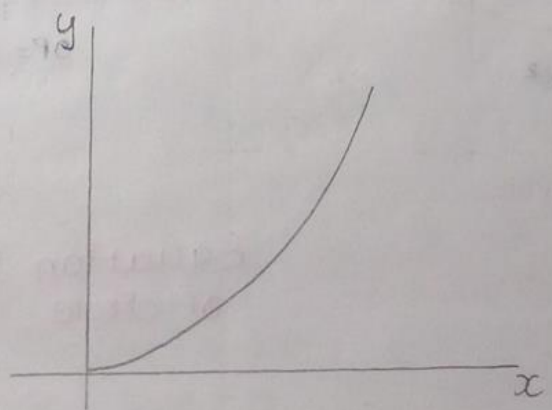
$a = 2$
$b = 3$



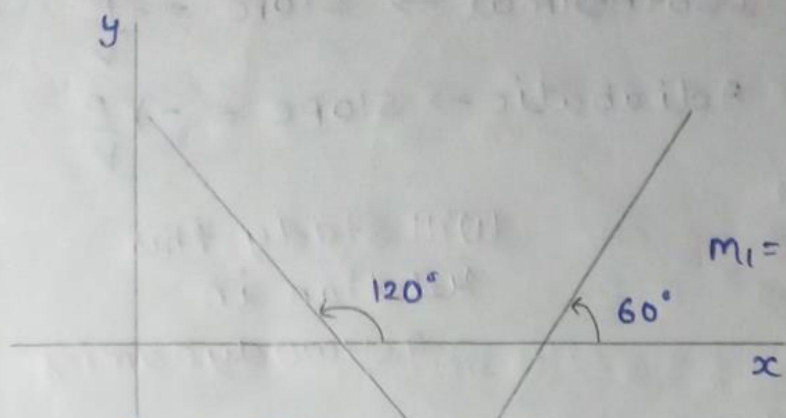
$$\frac{x^2}{4} + \frac{y^2}{4} = 1$$



Variation of slope



Magnitude of slope



$m_1 = \tan\theta = 60° = \sqrt{3}$

$m_2 = \tan(120°) = -\sqrt{3}$

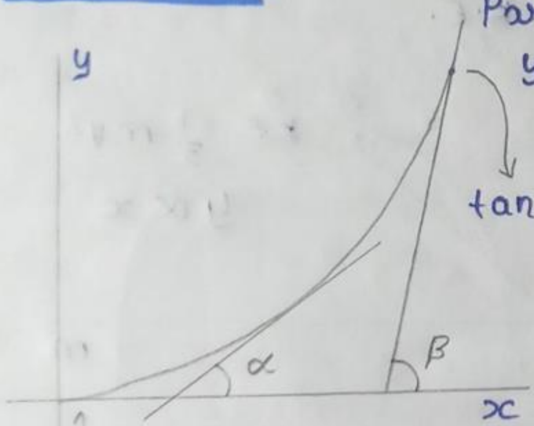
$|m_1| = |m_2|$

Magnitude

Parabolic

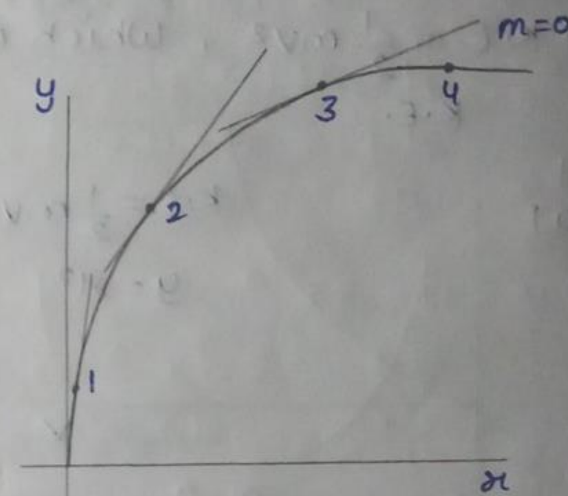
$y = x^2$

$\tan\beta = m$

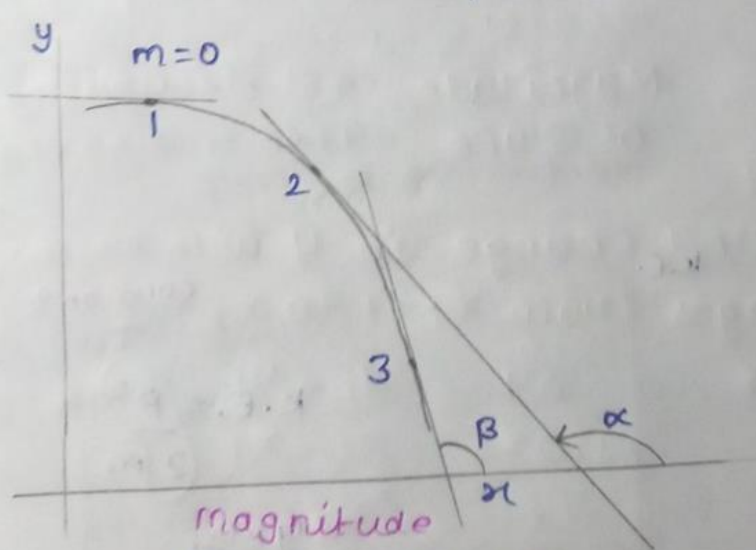


$m = 0$ $\quad$ $m = \tan\alpha$

magnitude of slope ↑



magnitude of slope decreasing

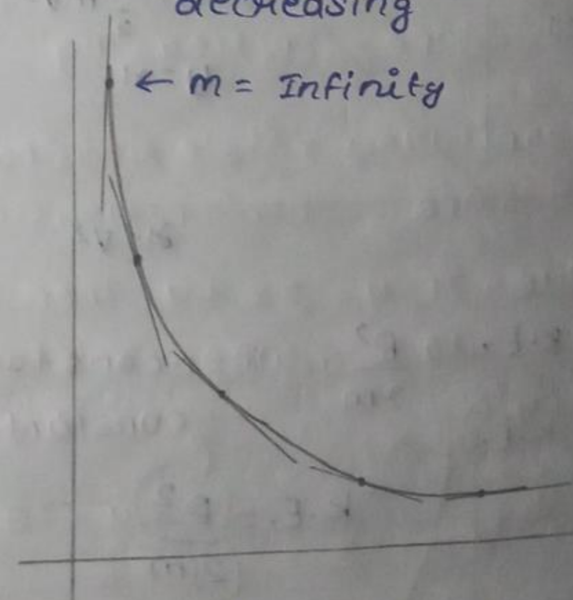


magnitude

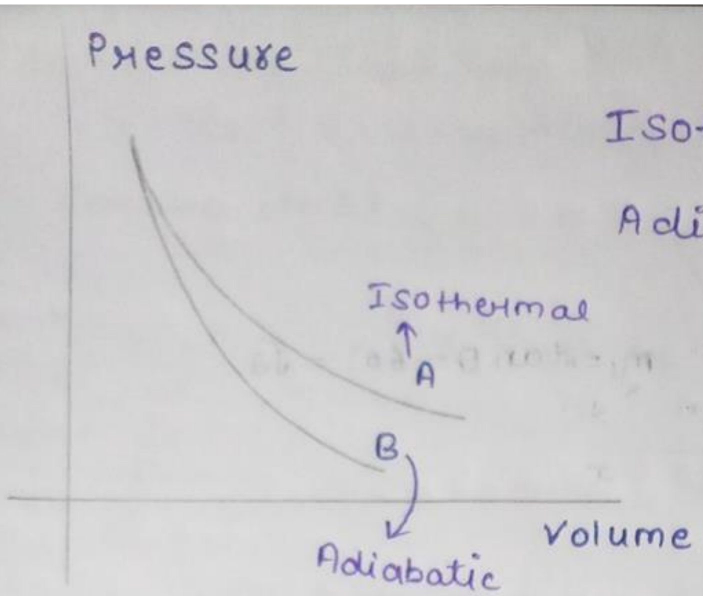
$m_1 = 0$ $\quad$ $m_3 > m_2$

→ Increasing magnitude of slope

← m = Infinity



magnitude of slope = ↓



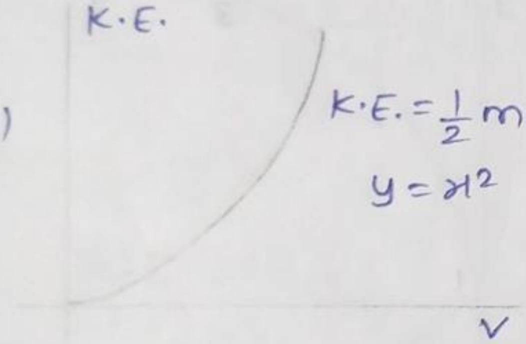
Isothermal → slope $= -\frac{P}{V}$

Adiabatic → slope $= -\gamma \frac{P}{V}$

Will study this relation in thermodynamics

$K.E = \frac{1}{2} mv^2$, which of the following is correct?

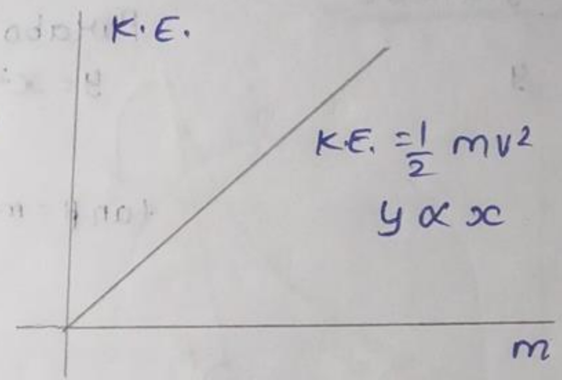
a)



$K.E. = \frac{1}{2} mv^2$

$y = x^2$

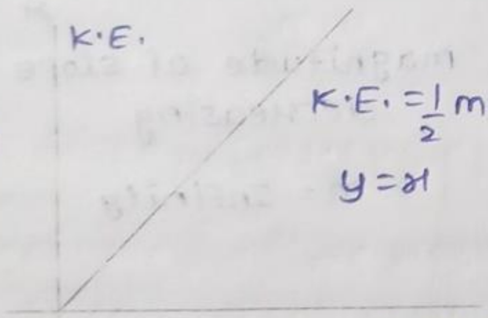
b)



$K.E. = \frac{1}{2} mv^2$

$y \propto x$

c)



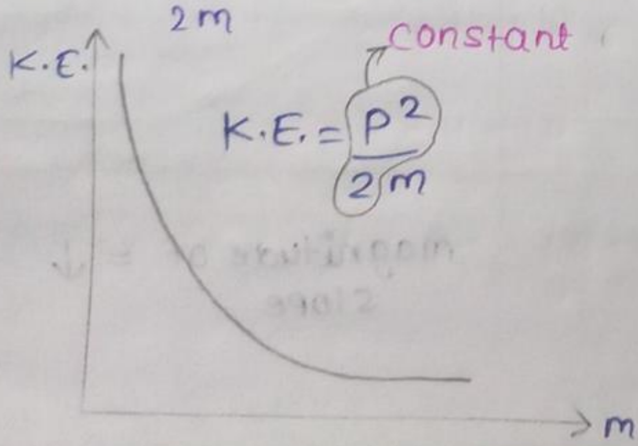
$K.E. = \frac{1}{2} mv^2$

$y = x$

$x = v^2$

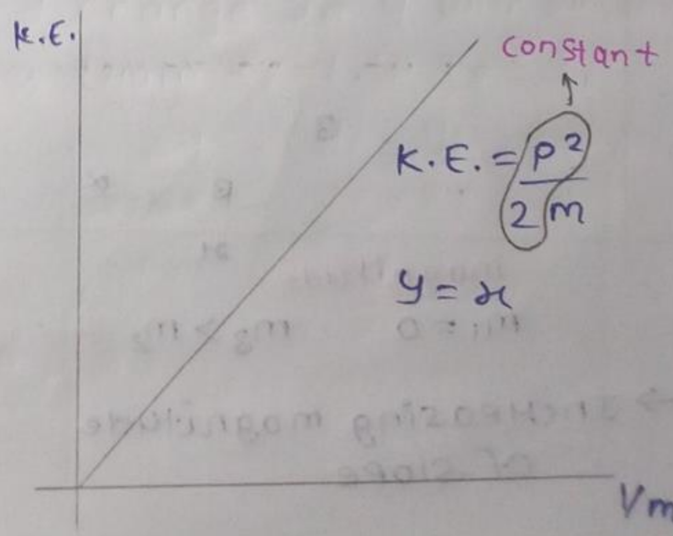
(d) All correct

$K.E. = \frac{P^2}{2m}$ P = constant



constant

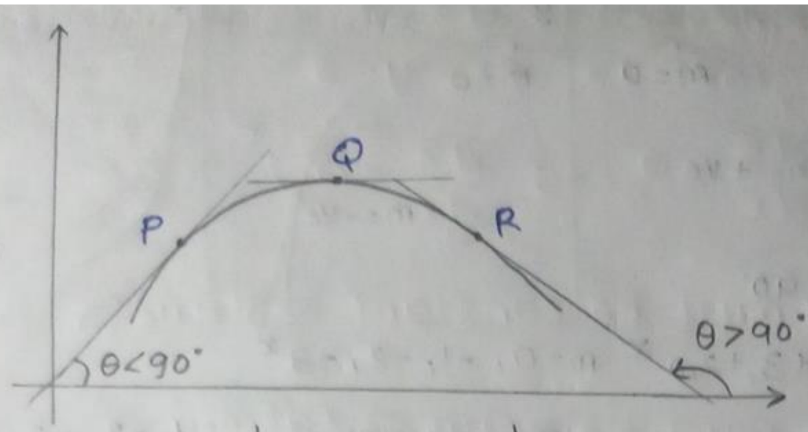
$K.E. = \frac{P^2}{2m}$



constant

$K.E. = \frac{P^2}{2m}$

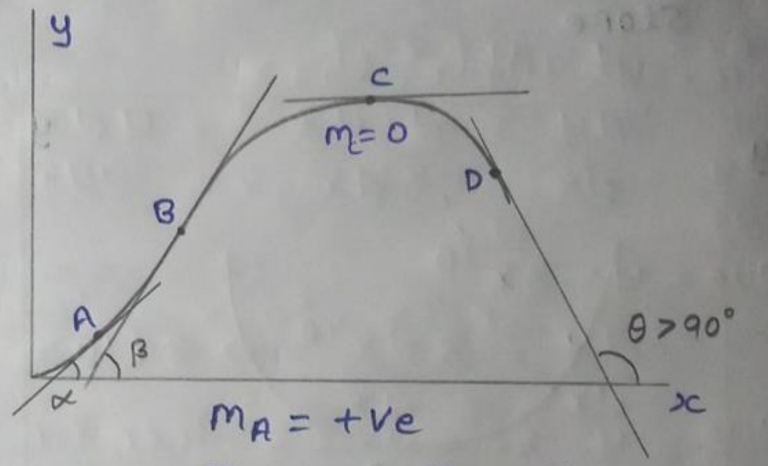
$y = x$



$m_P = \tan\theta$ | $m_Q = 0$ | $m_P = -ve$

$m_P = +ve$

Que- Match the matrix



Point	Slope
A	Zero
B	Negative
C	Maximum
D	Positive

$A \to +ve$

$B \to (+ve)$ maximum

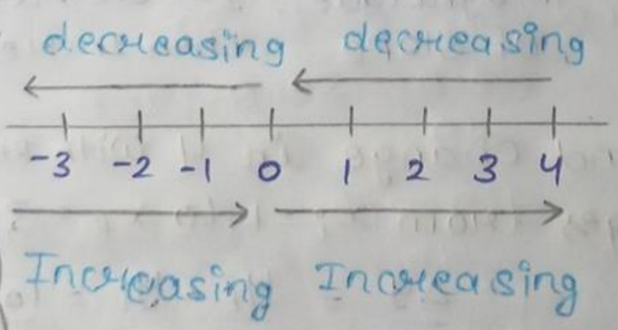
$C \to$ zero

$D \to -ve$

$m_A = +ve$

$m_B = +ve$ (max)

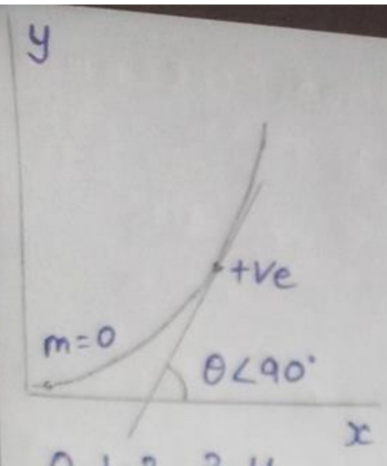
$m_D = -ve$



-ve ko

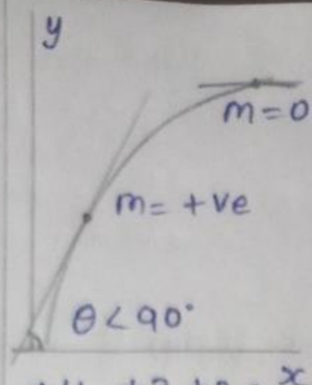
include kar

rahe ho



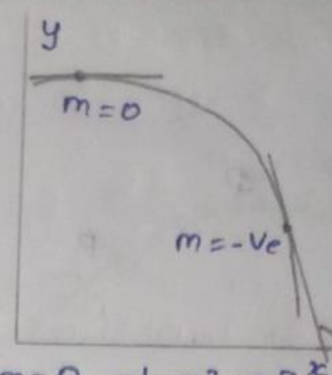
0, 1, 2, 3, 4 →

Slope - Increasing
↑
Magnitude of Slope



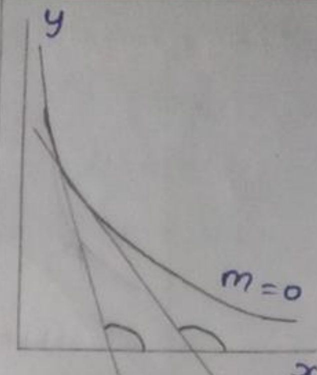
+4, +3, +2, 0

Slope → decreasing
↑
Magnitude of slope



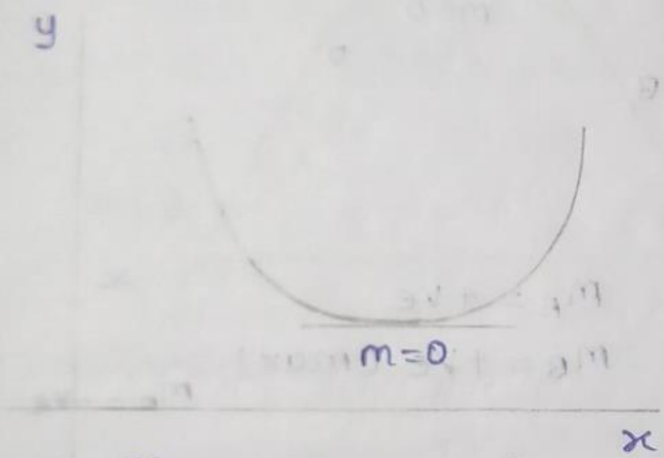
m = 0, -1, -2, -3

Slope - decreasing
Magnitude of slope
↑
Increasing



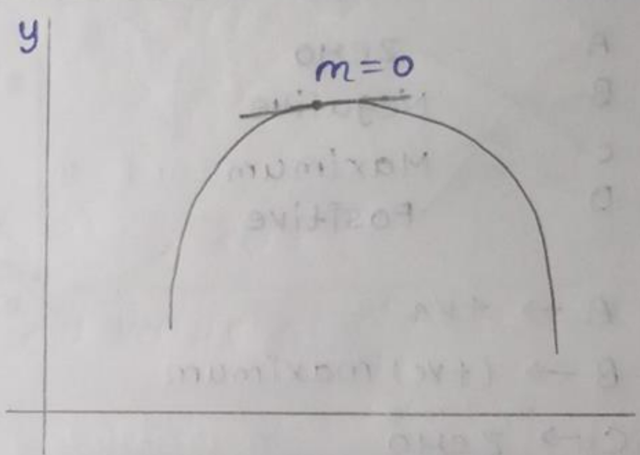
-4, -3, -2, 0

Slope - Increasing
Magnitude of slope
↑
Decreasing



Slope - Increasing

Magnitude of slope = I^{st} decreasing then Increasing



Slope - Decreasing

Magnitude of slope = I^{st} decreasing then Increasing

Que - If $y = 2x + 4$ then find change in y with respect to x, when x changes from $x_i = 1$ to $x_f = 3$

$(y_i)_{at\ x_i = 1} = 2\times1 + 4 = 6$

$(y_f)_{at\ x_f = 3} = 2\times3 + 4 = 10$

Change in $y = y_f - y_i = 10 - 6$
$= 4$

$$\frac{\Delta y}{\Delta x} = \frac{y_f - y_i}{x_f - x_i} = \frac{10-6}{3-1}$$

$$= \frac{4}{2} = 2$$

Que- If $y = 2x+4$ then find change in y w.r.t x at $x=4$ to $x = 4.00001$

$$\frac{\Delta y}{\Delta x} = \frac{y_f - y_i}{x_f - x_i}$$ (question is incomplete)

$$(\Delta x \to 0)\ \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = \frac{d}{dx} y$$

↑ very small change — ↑ Differentiation

$\frac{dy}{dx}$ = Rate of change in $\underline{y}$ w.r.t at x

$\frac{dP}{dx}$ = Rate of change in $\underline{P}$ w.r.t at x

Differentiation

Rule-1 → $y = x^n$ ($n \to$ power)

$$\frac{dy}{dx} = \frac{dx^n}{dx} = nx^{n-1}$$

Ex- $y = x^4$

$$\frac{dy}{dx} = \frac{dx^4}{dx} = 4x^{4-1} = 4x^3$$

② $y = \sqrt{x} = x^{1/2}$

$$\frac{dy}{dx} = \frac{dx^{1/2}}{dx} \Rightarrow \frac{1}{2} x^{1/2 - 1} \Rightarrow \frac{1}{2} x^{-1/2}$$

③ $y = x^{3/2}$

$$\frac{dy}{dx} = \frac{dx^{3/2}}{dx} \Rightarrow \frac{3}{2} x^{3/2 - 1}$$

$$= \frac{3}{2} x^{1/2}$$

④ $y = x^{10}$

$$\frac{dy}{dx} = \frac{dx^{10}}{dx} \Rightarrow 10x^9$$

⑤ $y = x^{-1/2}$

$$\frac{dy}{dx} = -\frac{1}{2} x^{-1/2 - 1}$$

$$-\frac{1}{2} x^{-3/2}$$

Rule-2 Constant Rule

If y is constant → Ex - $y = 5$

$\frac{dy}{dx} = 0$; $\frac{d\sin 30^\circ}{d\theta} = 0$; $\frac{d5}{dx} = 0$; $\frac{d\pi}{dx} = 0$; $\frac{d(\sin^2\theta + \cos^2\theta)}{d\theta} = 0$

If $\left(\frac{dy}{dx}\right) = 0$, then y must be constant → True

If $\left(\frac{dy}{dx}\right) = 0$, then y must be zero → False

Rule-3 - If any constant is multiplied with x, then it comes out from differentiate

Ex - $y = 4[x^2]$

$$\frac{dy}{dx} = \frac{d(4x^2)}{dx} = 4\frac{dx^2}{dx} \Rightarrow 4(2x^{2-1}) \Rightarrow 8x$$

Que - $y = 3x^{10}$

$$\frac{dy}{dx} = 3\frac{dx^{10}}{dx} \Rightarrow 3(10x^9) \Rightarrow 30x^9$$

Extra Point (MR Ratta)

$\frac{d\sin\theta}{d\theta} = \cos\theta$

$\frac{d\cos\theta}{d\theta} = -\sin\theta$

$\frac{d\tan\theta}{d\theta} = \sec^2\theta$

$\frac{d\cot\theta}{d\theta} = -\operatorname{cosec}^2\theta$

$\frac{d\sec\theta}{d\theta} = \sec\theta \cdot \tan\theta$

$\frac{d\operatorname{cosec}\theta}{d\theta} = -\operatorname{cosec}\theta \cdot \cot\theta$

Note → $y = e^2$

$\frac{dy}{dx} = \frac{de^2}{dx} = e^2$ ✗

0 ✓

e^2 is a constant value
So its differentiation is 0

$$\left[\begin{array}{l} \frac{d(e^x)}{dx} = e^x \\ \frac{d\log_e x}{dx} = \frac{d\ln x}{dx} = \frac{1}{x} \end{array}\right]$$

$y = x^2 + \sin x$	$y = x^2 - \sin(x)$	$y = x^2 \cdot \sin x$	$y = \sin x / x^2$
$y = A + B$	$y = A - B$	$y = A \cdot B$	$y = A/B$
$\frac{dy}{dx} = \frac{dA}{dx} + \frac{dB}{dx}$	$\frac{dy}{dx} = \frac{dA}{dx} - \frac{dB}{dx}$	$\frac{dy}{dx} = B\frac{dA}{dx} + A\frac{dB}{dx}$	$\frac{dy}{dx} = \frac{\frac{dA}{dx} \times B - A\frac{dB}{dx}}{B^2}$
$\frac{dy}{dx} = 2x + \cos\theta$	$\frac{dy}{dx} = 2x - \cos(x)$	$\frac{dy}{dx} = \frac{dx^2}{dx}\sin(x) + x^2 \frac{d\sin(x)}{dx}$ $\Rightarrow 2x\sin x + x^2\cos x$	$\frac{dy}{dx} = \frac{x^2 \frac{d\sin(x)}{dx} - \sin(x)\frac{dx^2}{dx}}{x^4}$ $\frac{dy}{dx} = \frac{x^2\cos x - \sin x\,(2x)}{x^4}$

Que - $y = 3x^2 - 4 + \sin x + e^x + 4\log x$

$$\frac{dy}{dx} = \frac{d(3x^2)}{dx} - \frac{d4}{dx} + \frac{d\sin(x)}{dx} + \frac{de^x}{dx} + \frac{4\,d\log x}{dx}$$

$$\rightarrow 3[2x] - 0 + \cos x + e^x + 4/x$$

$$\rightarrow 6x + \cos x + e^x + 4/x$$

Que - $y = \cos x - 4/x^2$

$$\frac{dy}{dx} = -\sin x - 4[-2/x^3] \rightarrow \left[-\sin + \frac{8}{x^3}\right]$$

Que - $y = e^x \sin(x)$

$$\frac{dy}{dx} = \frac{de^x}{dx}\sin x + \frac{d\sin x}{dx}e^x$$

$$\frac{dy}{dx} = e^x \sin x + \cos x\, e^x$$

Que - Find differentiation of $\rightarrow$ $y = \frac{x^2}{x^2+1}$

$$\frac{dy}{dx} = \frac{\frac{dx^2}{dx}(x^2+1) - x^2\frac{d(x^2+1)}{dx}}{(x^2+1)^2}$$

$$= \frac{2x(x^2+1) - x^2(2x+0)}{(x^2+1)^2}$$

$$y = \frac{x^2+1}{x^2} = \frac{x^2}{x^2} + \frac{1}{x^2}$$

$$y = 1 + \frac{1}{x^2}$$

$$\frac{dy}{dx} = 0 - \frac{2}{x^3} = -\frac{2}{x^3}$$

Que - $y = x^2 \cdot x^4$

$$\frac{dy}{dx} = 2x(x^4) + x^2(4x^3)$$

$$= 2x^5 + 4x^5$$

$$= 6x^5$$

$$y = x^6$$

$$\frac{dy}{dx} = 6x^5$$

Que - $y = \tan x . \log x$

$$\frac{dy}{dx} = \frac{d(\tan x)}{dx}\log x + \tan(x)\frac{d\log(x)}{dx}$$

$$= \sec^2 x \cdot \log x + \frac{\tan x}{x}$$

Que - $y = x^2 - 4x + 3$, then find value of $\frac{dy}{dx}$ at $x=2$

$$\frac{dy}{dx} = 2x - 4\frac{dx}{dx} \rightarrow \frac{dy}{dx} = 2x - 4 \rightarrow \left|\frac{dy}{dx}\right|_{x=2} \Rightarrow 2\times 2 - 4$$

$$= 0$$

Que - If $y = \sin\theta$, then find $\left(\frac{dy}{d\theta}\right)$ at $\theta = 30^\circ$

$$y = \sin\theta$$

$$\frac{dy}{d\theta} = \cos\theta$$

$$\left(\frac{dy}{d\theta}\right)_{\theta=30^\circ} = \cos 30^\circ = \frac{\sqrt{3}}{2}$$

Que - If $y = e^x$ then find $\frac{dy}{dx}$ at $x = 2$

$y = e^x \rightarrow \frac{dy}{dx} = e^x \rightarrow \left|\frac{dy}{dx}\right|_{x=2} = e^x$

Que- If $y = x^2 e^x$ then find $\frac{dy}{dx}$

$y = x^2 . e^x \rightarrow \frac{dy}{dx} = \frac{dx^2}{dx} e^x + x^2 \frac{de^x}{dx}$

$\rightarrow 2x e^x + x^2 e^x$

Que- If $y = t(t^2-2)$ then find $\left(\frac{dy}{dt}\right)$

$y = t(t^2-2)$

$y = t^3 - 2t$

$\frac{dy}{dt} = [3t^2 - 2]$

Que- If $y = x^2 - 4x$ then find y, when rate of change in y w.r.t x is zero

$y = x^2 - 4x \rightarrow \frac{dy}{dx} = 2x - 4 = 0$

$2x = 4$

$\rightarrow [x = 2]$

$y = (2)^2 - 4 \times 2 \Rightarrow 4 - 8 = -4$

Que- $y = \sqrt{x} + \frac{1}{\sqrt{x}}$

$y = x^{1/2} + x^{-1/2}$

$\frac{dy}{dx} = \frac{1}{2} x^{1/2 - 1} - \frac{1}{2} x^{-1/2 - 1}$

$= \frac{1}{2} x^{-1/2} - \frac{1}{2} x^{-3/2}$

Que- $y = \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2$

$y = x + \frac{1}{x} + 2\sqrt{x} \cdot \frac{1}{\sqrt{x}}$

$y = x + \frac{1}{x}$

$\frac{dy}{dx} = \left(1 - \frac{1}{x^2}\right)$

Double Differentiation

$y = 4x^5$ find $\frac{d^2y}{dx^2}$

$$\frac{dy}{dx} = 4\left[\frac{dx^5}{dx}\right] \Rightarrow 20x^4 \rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d(20x^4)}{dx}$$

$$\rightarrow 20\frac{dx^4}{dx} = 80x^3$$

Que - $y = A\sin\theta$ find $\frac{d^2y}{d\theta^2}$, find $\frac{d^2y}{d\theta^2}$

$$\frac{dy}{d\theta} = A\cos\theta \quad \rightarrow \frac{d}{d\theta}\left(\frac{dy}{d\theta}\right) = \frac{d^2y}{d\theta^2} = A[-\sin\theta]$$
$$= -A\sin\theta$$

Que - $y = 1x^5 + 2x^4 + 3x^3 + 4x^2 + 5x$

$$\frac{dy}{dx} = 5x^4 + 8x^3 + 9x^2 + 8x + 5$$

$$\frac{d^2y}{dx^2} = 20x^3 + 24x^2 + 18x + 8 + 0$$

$$\frac{d^3y}{dx^3} = 60x^2 + 48x + 18 + 0$$

$$\frac{d^4y}{dx^4} = 120x + 48 + 0 \rightarrow \frac{d^5y}{dx^5} = 120 \rightarrow \boxed{\frac{d^6y}{dx^6} = 0}$$

Que - $y = e^x$

$$\frac{dy}{dx} = e^x$$

$$\frac{d^2y}{dx^2} = e^x$$

$$\frac{d^3y}{dx^3} = e^x$$

$$\frac{d^4y}{dx^4} = e^x$$

$y = \sin x$

$$\frac{dy}{dx} = \cos x$$

$$\frac{d^2y}{dx^2} = -\sin x$$

$$\frac{d^3y}{dx^3} = -\cos x$$

$$\frac{d^4y}{dx^4} = \sin x$$

$$\frac{d^5y}{dx^5} = +\cos x \quad \frac{d^6y}{dx^6} = -\sin x$$

Function of Function

$y = \sin x \times e^x$ → Product rule

$y = \log x + x^5$ → addition rule

outside ↓ ← Inside

$$\left[\begin{array}{l} y = \sin(e^x) \\ y = \sin(x^2 + 4x) \end{array}\right] \rightarrow \text{Function of function}$$

↑ outside ↑ Inside

$\frac{dy}{dx}$ = (differentiate of outer function keep inside as it is) × (differentiate of inner function)

(i) $y = \sin(3x+4) \rightarrow \frac{dy}{dx} = \cos(4x+3) \times 3$

(ii) $y = \cos(e^x) \rightarrow \frac{dy}{dx} = -\sin(e^x) \times e^x$

Que - Find differentiation of function
$y = \sin(2x^2 - 6x)$

$$\frac{dy}{dx} = \cos(2x^2 - 6x) \times (4x - 6)$$

(e) $\left|\frac{dy}{dx}\right|$ = Magnitude of rate of change in y w.r.t. x

(f) $\frac{d|y|}{dx}$ = The rate of change in magnitude of w.r.t. x

Que- If velocity of object $v = 3t\hat{i} + 4t\hat{j}$ then find

(i) The rate of change in velocity

(ii) The magnitude of rate of change in velocity

(iii) The rate of change in magnitude of velocity

(i) $\frac{dv}{dt} = 3\hat{i} + 4\hat{j}$

(ii) $\left|\frac{dv}{dt}\right| = \sqrt{(3)^2 + (4)^2}$

$= 5$

$V = 3t\hat{i} + 4t\hat{j}$

$|V| = \sqrt{(3t)^2 + (4t)^2}$

$= \sqrt{9t^2 + 16t^2}$

$V = 5t \rightarrow \frac{d|V|}{dt} \Rightarrow \frac{5\,dt}{dt}$

$= 5$

Que -

1. $y = e^{(\sin x)} \rightarrow \frac{dy}{dx} = e^{\sin x} \times \cos x$

2. $y = \cos(x^2 + 4x) \rightarrow \frac{dy}{dx} = -\sin(x^2 + 4x) \times (2x + 4)$

3. $y = 4\sin(8x) \rightarrow \frac{dy}{dx} = 4\cos(8x) \times 8$

$= 32\cos(8x)$

Que - Find differentiation

1. $y = e^{-\alpha x} \rightarrow y = e^{(-\alpha x)} \rightarrow \frac{dy}{dx} = e^{-\alpha x} \times \frac{d(-\alpha x)}{dx}$

$= e^{-\alpha x}(-\alpha)$

$= -\alpha e^{-\alpha x}$

2. $y = e^{(4x-3)} \rightarrow \frac{dy}{dx} = 4e^{4x-3}$

3. $y = e^{(x^2+2)} \rightarrow \frac{dy}{dx} = e^{(x^2+2)} \times 2x$

Que - $I = I_0 e^{-t/RC}$

$$\frac{dI}{dt} = \frac{I_0 \, d e^{-t/RC}}{dt}$$

$$= I_0 e^{-t/RC} \times \left(\frac{-1}{RC} \frac{dt}{dt}\right)$$

$$\frac{dI}{dt} = \frac{-I_0}{RC} e^{-t/RC}$$

Que - $Q = Q_0 (1 - e^{-t/\tau})$

$$\frac{dQ}{dt} = Q_0 \left[\frac{d1}{dt} - \frac{d e^{-t/\tau}}{dt}\right]$$

$$\Rightarrow Q_0 \left[0 - e^{-t/\tau} \times \left(-\frac{1}{\tau}\right) \frac{dt}{dt}\right]$$

$$\Rightarrow Q_0 \left[\frac{1}{\tau} e^{-t/\tau}\right]$$

$$\Rightarrow \frac{Q_0}{\tau} e^{-t/\tau}$$

Que-

(i) $y = e^{4x} \rightarrow \frac{dy}{dx} = 4e^{4x}$

(ii) $y = e^{(3x+5)} \rightarrow \frac{dy}{dx} = 3e^{3x+5}$

(iii) $y = \sin 2x \rightarrow \frac{dy}{dx} = \cos(2x) \times 2$

$$= 2\cos(2x)$$

(iv) $y = \ln(3x+4) \rightarrow \frac{dy}{dx} = \frac{1}{(3x+4)} \times \frac{d(3x+4)}{dx}$

$$\frac{dy}{dx} = \frac{3}{3x+4}$$

(v) $y = \sin(\cos x) \rightarrow y = \cos(\cos x) \times -\sin x$

$$= -\sin x \cos(\cos x)$$

$y = A \sin(\omega t - kx) \rightarrow A, \omega, k$ are constant

$$\frac{dy}{dx} = A\cos(\omega t - kx) \times \frac{d(\omega t - kx)}{dx}$$

$$= A\cos(\omega t - kx)(-k)$$

$$\frac{dy}{dx} = -Ak\cos(\omega t - kx)$$

$$\frac{d^2y}{dx^2} = -Ak[-\sin(\omega t - kx) \times (-k)]$$

$$= -Ak^2 \sin(\omega t - kx)]$$

$$y = A\sin(\omega t - kx)$$

$$\frac{dy}{dt} = A\cos(\omega t - kx)\frac{d(\omega t - kx)}{dt}$$

$$A\cos(\omega t - kx) \times \omega$$

$$\frac{dy}{dt} = A\omega\cos(\omega t - kx)$$

$$\frac{d^2y}{dt^2} = A\omega[-\sin(\omega t - kx)] \times \omega$$

$$\frac{d^2y}{dt^2} = -A\omega^2 \sin(\omega t - kx)$$

Que- Find differentiation of

$y = \sin^2 x \xleftrightarrow[\text{same}]{} y = (\sin x)^2$

$$y = (\sin x)^2$$

$$\frac{dy}{dx} = 2(\sin x)^{2-1} \times \frac{d\sin x}{dx}$$

$$= 2\sin x \times \cos x$$

$$\frac{dy}{dx} = \sin(2x)$$

$$y = \sin(x^2)$$

$$\frac{dy}{dx} = \cos(x^2) \times \frac{dx^2}{dx}$$

$$= \cos(x^2) \times 2x$$

Que- Find differentiation of

(i) $y = \log(3x+4) \rightarrow \frac{dy}{dx} = \frac{1}{3x+4} \times 3 = \frac{3}{3x+4}$

(ii) $y = (4x+3)^2 \rightarrow y = (4x+3)^2$

$$\frac{dy}{dx} = 2(4x+3)^{2-1} \times \frac{d(4x+3)}{dx}$$

$$= 2(4x+3) \times 4$$

$$= 8(4x+3)$$

$$= 32x + 24$$

Que- Find differentiation of $y = (x^4-1)^{50}$

$$\frac{dy}{dx} = 50(x^4-1)^{50-1} \times \frac{d(x^4-1)}{dx}$$

$$= 50(x^4-1)^{49} \times (4x^3-0)$$

$$= 200x^3(x^4-1)^{49}$$

Que- $y = \sqrt{x^2+4x}$

$y = (x^2+4x)^{1/2} \rightarrow \frac{dy}{dx} = \frac{1}{2}(x^2+4x)^{1/2-1} \times \frac{d(x^2+4x)}{dx}$

$$= \frac{1}{2\sqrt{x^2+4x}} \times (2x+4)$$

Que- $y = \frac{1}{(2x+3)}$

$y = (2x+3)^{-1} \rightarrow \frac{dy}{dx} = -1(2x+3)^{-1-1} \times \frac{d(2x+3)}{dx}$

$$= \frac{-1\times 2}{(2x+3)^2} \Rightarrow \frac{-2}{(2x+3)^2}$$

Que- If $y = x^2$ then find $\frac{dy}{dt}$ where x depends on 't'

MR*

$y = x^2$

$\frac{dy}{dx} = 2x$

$\frac{dy}{dt} = 2x\frac{dx}{dt}$

$y = x^2$

diff. w.r.t. 't'

$\frac{dy}{dt} = \frac{dx^2}{dt} \times \frac{dx}{dx}$

$\frac{dx^2}{dx} \times \frac{dx}{dt}$

$$\boxed{\frac{dy}{dt} = 2x\frac{dx}{dt}}$$

Que- If $V = \frac{4}{3}\pi R^3$, find Rate of change in volume w.r.t time $\left(\frac{dV}{dt}\right)$

$$V = \frac{4}{3}\pi R^3 \rightarrow \frac{dV}{dR} = \frac{4\pi}{3}[3R^2]$$

$$\frac{dV}{dt} = 4\pi R^2\left(\frac{dR}{dt}\right)$$

Que- If radius of circle is increasing $\frac{1}{\pi}$ m/s then find rate of change in area when radius is 4m

$$A = \pi R^2$$

$$\frac{dA}{dR} = \pi 2R \rightarrow \frac{dA}{dt} = 2\pi R \frac{dR}{dt}$$

$$\rightarrow \frac{dA}{dt} = 2\pi \times 4\left[\frac{1}{\pi}\right] = 8 \ m^2/sec$$

Partial Differentiation

$$t = f(x, y, z)$$

$\left(\frac{dt}{dx}\right)_{y,z \rightarrow \text{constant}}$ = Partial diffn of t w.r.t x then y & z remains constant

$\left(\frac{dt}{dy}\right)_{x,z \rightarrow \text{constant}}$ = Partial diffn of t w.r.t y

Que- If $V = x^2y + y^2z + z^2x$; find $\frac{dV}{dx}$, $\frac{dV}{dy}$, $\frac{dV}{dz}$

$$\left(\frac{dV}{dx}\right)_{y,z = \text{constant}} = \frac{d(x^2y + y^2z + z^2x)}{dx}$$

$$= y\frac{dx^2}{dx} + 0 + z^2\frac{dx}{dx}$$

$$= (2xy + z^2)$$

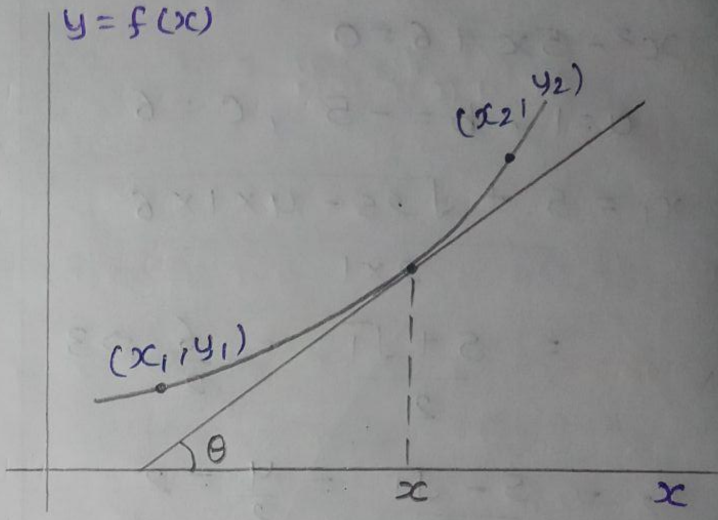
$$\frac{dv}{dy} = x^2\frac{dy}{dy} + z\frac{dy^2}{dx} + 0$$

$$= x^2 + 2yz$$

$$\left(\frac{dv}{dz}\right)_{x,y \rightarrow \text{constant}} = 0 + (y^2 + 2zx)$$

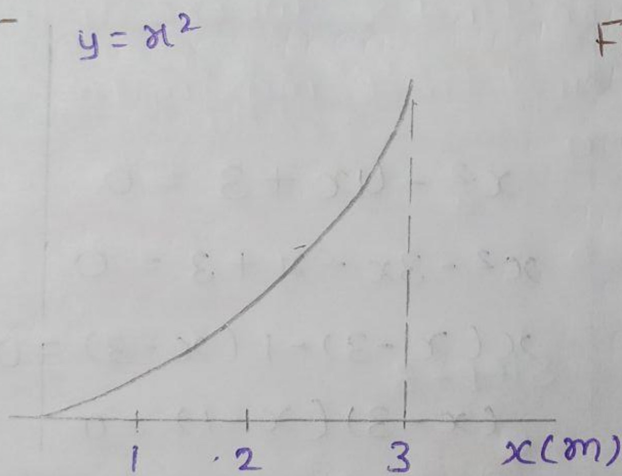
Slope of graph at a point

Slope b/w two point $= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$



Slope $= \tan\theta = \frac{dy}{dx}$ → diff. of 'y' w.r.t 'x'

Que-



Find slope of graph at

$x = 3m$

$y = x^2$

$\left|\frac{dy}{dx}\right| = 2x$

→ Slope $= 2x = 6$

Quadratic Equation

$ax^2 + bx + c = 0$, where a, b, c are constant

$$* \; x_1 = \frac{-(b) + \sqrt{b^2 - 4ac}}{2a}$$

$$* \; x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

→ Roots of the equation

$$x_1 + x_2 = -\frac{2b}{2a} = -\frac{b}{a}$$

$$x_1 \cdot x_2 = \frac{c}{a}$$

$$x^2 - 4x + 8 = 0$$

$$a = 1, \; b = -4, \; c = 8$$

Que- Find root of the equation

$$x^2 - 5x + 6 = 0$$

$$a = 1, \; b = -5, \; c = 6$$

$$x_1 = \frac{5 + \sqrt{25 - 4 \times 1 \times 6}}{2 \times 1}$$

$$= \frac{5 + \sqrt{1}}{2} = \frac{6}{2} \Rightarrow 3$$

$$x_2 = \frac{5 - \sqrt{1}}{2} = \frac{4}{2} = 2$$

$$x_1 x_2 = 6 \rightarrow \frac{c}{a} = \frac{6}{1}$$

$$x_1 + x_2 = 5 \rightarrow -\frac{b}{a} = -\left(\frac{-5}{1}\right) = +5$$

$$x^2 - 5x + 6 = 0$$

$$x^2 - 3x - 2x + 6 = 0$$

$$x(x-3) - 2(x-3) = 0$$

$$(x-3)(x-2) = 0$$

$$x = 3 \;/\; x = 2$$

Que- $x^2 - 4x = 0$

$$x[x-4] = 0$$

$$x = 0 \;/\; x = 4$$

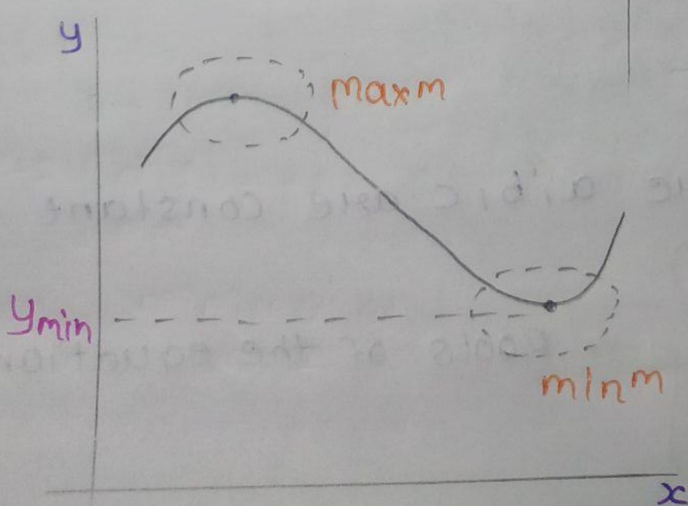
$$x^2 - 4x + 3 = 0$$

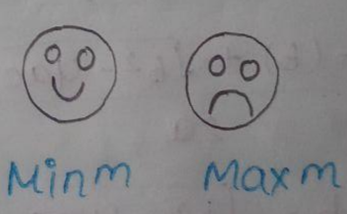
$$x^2 - 3x - x + 3 = 0$$

$$x(x-3) - 1(x-3) = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \;/\; x = 1$$



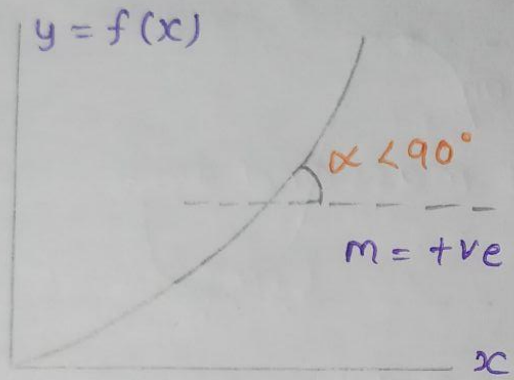


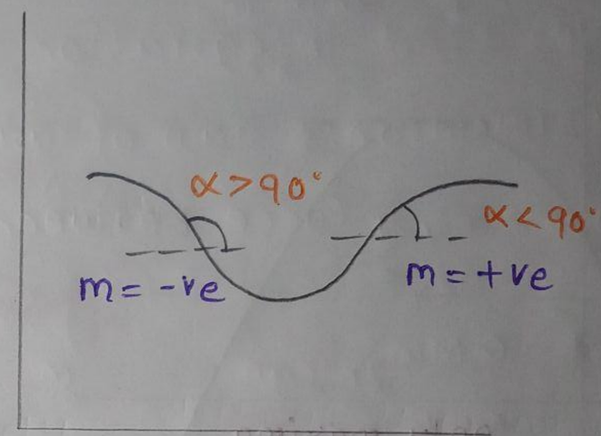
Double differentiation of 'y'

$\left(\frac{dy}{dx}\right)$ = differentiation of 'y' w.r.t = (slope) at a point
= Rate of change in y w.r.t. x

$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$ = Rate of change in $\frac{dy}{dx}$ w.r.t. X

$\frac{d\,(\text{slope})}{dx}$ = Rate of change in slope w.r.t 'x'
= slope of slope

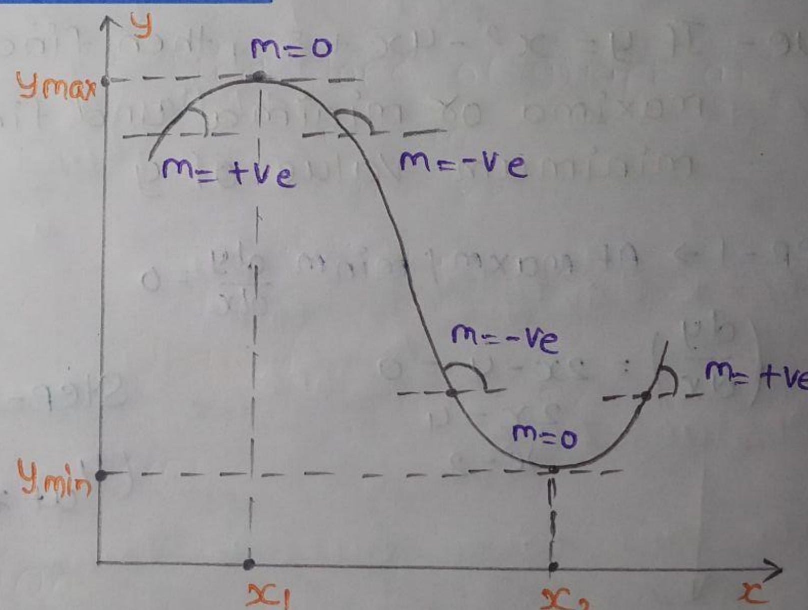




Maxima / Minima value of 'y'

* at x_1, y is maxima
* at x_2, y is minima

$$\boxed{\begin{array}{c}\text{Slope} = 0 \\ \frac{dy}{dx} = 0\end{array}}$$

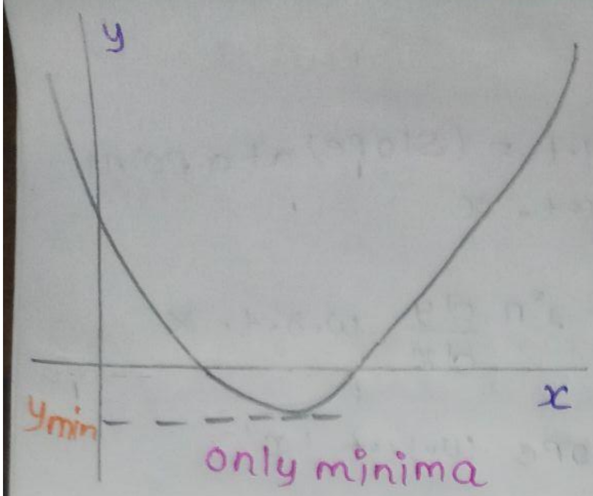


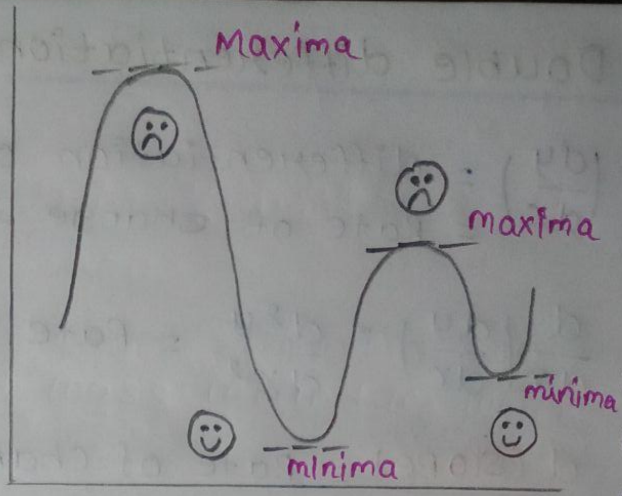
At maxima ☹

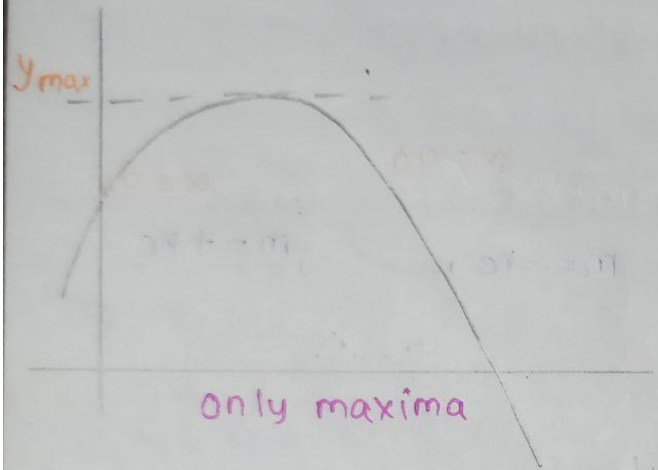
- Change in slope = -ve
- $\frac{d^2y}{dx^2} = -ve$

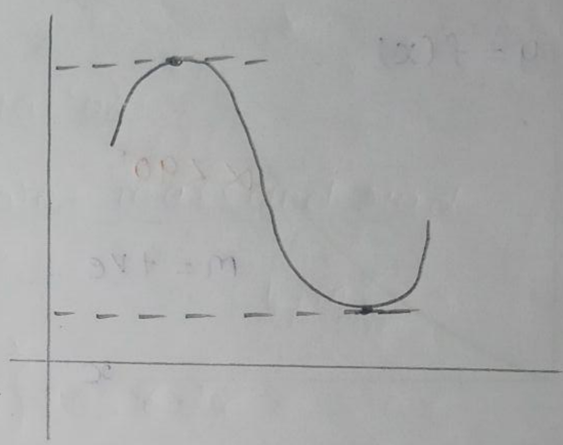
At minima ☺

- Change in slope = +ve
- $\frac{d^2y}{dx^2} = +ve$









Que- If $y = x^2 - 4x + 5$, then find x, where y will be maxima or minima and find maximum or minimum value of y

Step-1 → At maxm / minm $\frac{dy}{dx} = 0$

$$\left(\frac{dy}{dx}\right) = 2x - 4 = 0$$
$$2x = 4$$
$$x = 2$$

Step-2 $\left[\frac{d^2y}{dx^2}\right] = +2$

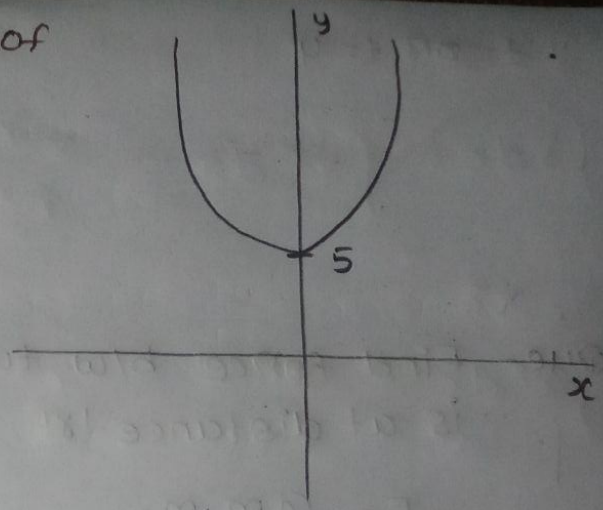
minima at $x = 2$

Step-3

$$(y_{at\, x=2})_{min.} \rightarrow (2)^2 - 4\times 2 + 5$$
$$= 4 - 8 + 5$$
$$= 9 - 8$$
$$y_{minm} = +1$$

Que - Find maxima and minima of

(i) $y = x^2 + 5$



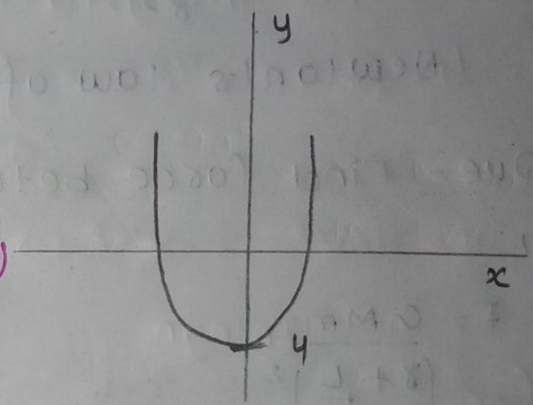
$\frac{dy}{dx} = 0$ (max^m / min^m)

$\frac{dy}{dx} = 2x = 0$

$x = 0$ → min^m

$\frac{dy}{dx} = +2$ $\qquad (y_{x=0}) = 5$

(ii) $y = x^2 - 8x$



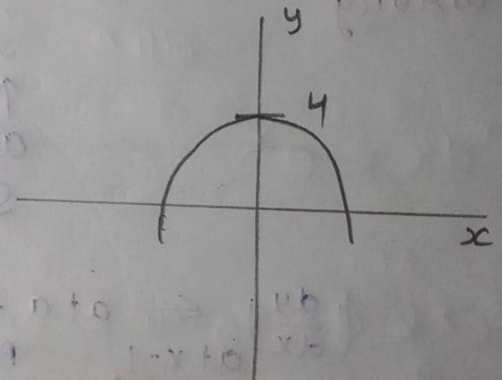
$\frac{dy}{dx} = 2x - 8 = 0$ (max^m / min^m)

$2x = 8$

$x = +4$ → at $x = 4$ (min^m)

$y_{min (x=4)} = (4)^2 - 8 \times 4$
$= 16 - 32$
$= -16$

(iii) $y = 4x - x^2$



At max^m / min^m $\left(\frac{dy}{dx} = 0\right)$

$\frac{dy}{dx} = 4 - 2x = 0$

$4 = 2x$

$x = 2$ → $y_{max} = 4 \times 2 - (2)^2$
$x=2 \quad = 8 - 4$

$y_{max} = +4$

$\frac{d^2y}{dx^2} = 0 - 2$ max^m

Que - If velocity $v = t^3 - 6t^2 + 12$, then find maxima and minima value of velocity

At max^m / min^m

$\frac{dv}{dt} = 0 =$ slope

$3t^2 - 12t + 0 = 0$

$3t(t - 4) = 0$

$t = 0 \,/\, t = 4$

$(y)_{t=0} = 12$

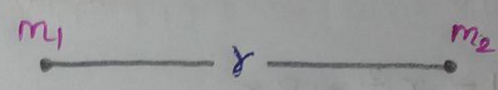
max^m

$(y)_{t=4} = (4)^3 - 6(4)^2 + 12$

min^m $= -20$

Que- Find force b/w two point mass m_1 and m_2 which is at distance 'r'

$$F = \frac{G m_1 m_2}{r^2}$$

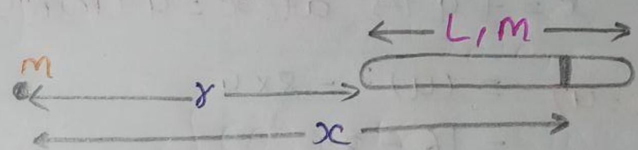


(Newton's law of gravitation)

Que- Find force between Point mass and rod

$$F = \frac{GMm}{\left(r + \frac{L}{2}\right)^2}$$

wrong

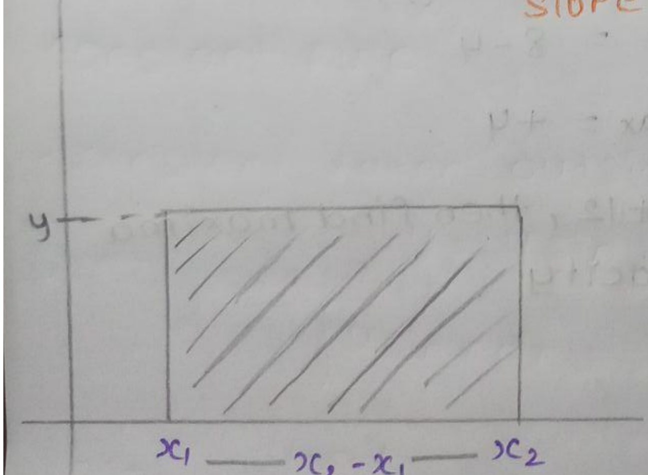


$$\int dF = \int \frac{Gm\,dm}{x^2}$$

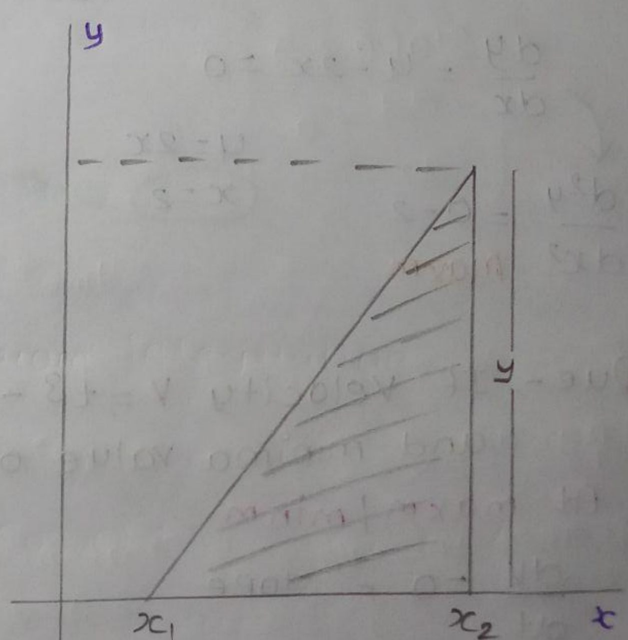
↑
addition of large no. of very small terms - Integration

$\left(\frac{dy}{dx}\right)_{at\ x=1}$ ← at a fixed Point

slope



Area $= (x_2 - x_1)y$

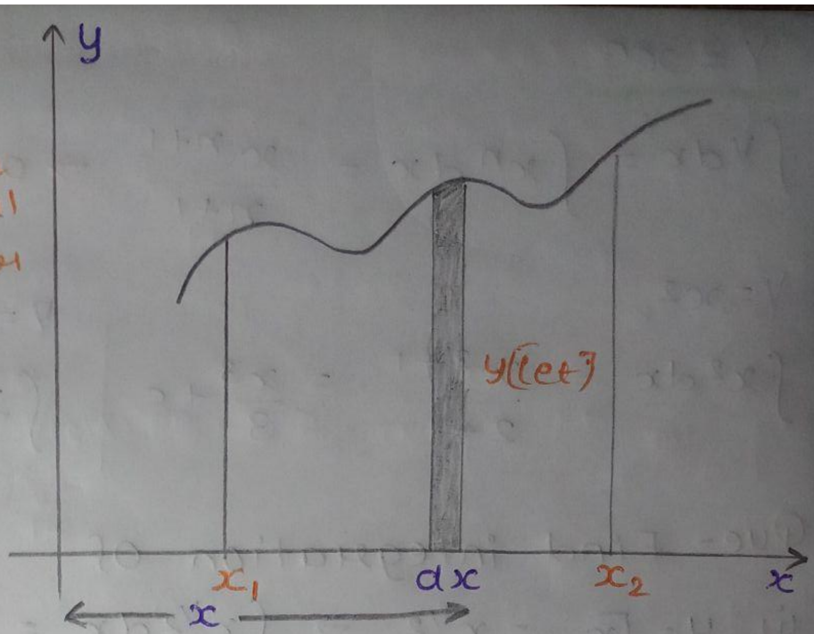


Area $= \frac{1}{2} \times$ Base $\times$ Height

$= \frac{1}{2} \times (x_2 - x_1)y$

$$\int dA = \int_{x_1}^{x_2} y\,dx$$

x_2 → upper limit
→ Integration in a interval
→ Area under curve
x_1 → Lower limit



Integration – $\int_{x_1}^{x_2} y\,dx$ → Area under the curve of y/x

Differentiation – $\frac{dy}{dx}$ → slope of y-x graph at a point

Integration

$\int y\,dx$ → Indefinite Integration

Definite Integration → $\int_{x_1}^{x_2} y\,dx$

$\int y\,dx = y' + c$.

- Integration is a inverse of Differentiation.

$\int \square\,dx$ = Integration of $\square$ w.r.t. 'x'

$\int y\,dx$ = Integ^n of y w.r.t. 'x'

$\int y\,dt$ = Integ^n of y w.r.t 't'

$Y = x^n$

$\int Y\,dx = \int x^n\,dx = \frac{x^{n+1}}{n+1}$ → only valid when $n \neq 1$

$Y = x^2$

$\int x^2 dx = \frac{x^{2+1}}{2+1} = \frac{x^3}{3} + c$

$Y = x^5$

$\int x^5 dx = \frac{x^{5+1}}{5+1} = \frac{x^6}{6} + c$

Que- Find integration of

(i) $y = \sqrt{x} = x^{1/2} \rightarrow \int x^{1/2} dx = \frac{x^{1/2+1}}{\frac{1}{2}+1} = \frac{x^{3/2}}{3/2} + c$

(ii) $y = 1/x^2 = x^{-2} \rightarrow \int x^{-2} dx = \frac{x^{-2+1}}{-2+1} = \frac{1}{-x^1} = -\frac{1}{x}$

Integn

$\int \sin x\,dx = -\cos x$

Diffn

Integn

$\int \cos x\,dx = \sin x$

Diffn

$\int e^x dx = e^x$

$\int \frac{1}{x} dx = \int x^{-1} dx = \log x$

$\int \sec^2 x\,dx = \tan x$

Constant Rule

- constant taken outside from integration.

$y = 4 \sin x$

$\int 4 \sin x\,dx = 4(-\cos x) + c$

Ex- $Y = 4x^4$

$\int 4x^4 dx \rightarrow 4\int x^4 dx \Rightarrow 4\frac{x^5}{5} + c$

Addition/Subtract Rule

$y = 4x^3 + \cos x + e^x$

$= \int (4x^3 + \cos x + e^x)\,dx$

$= \int 4x^3 dx + \int \cos(x)\,dx + \int e^x dx$

$= 4\frac{x^4}{4} + \sin(x) + e^x$

$= x^4 + \sin x + e^x$

$y = \frac{4}{x} - \sin x + x^3$

$= \int y\,dx = \int \frac{4}{x}dx - \int \sin x\,dx + \int x^3 dx$

$= 4\log x - (-\cos x) + \frac{x^{3+1}}{3+1} + c$

$= 4\log x + \cos x + \frac{x^4}{4} + c$

Que- $y = 4$

$\int y\,dx \rightarrow \int 4\,dx \rightarrow 4\int 1\,dx \Rightarrow 4\int x^0 dx$

$\Rightarrow 4\left[\frac{x^{0+1}}{0+1}\right] \Rightarrow 4x^1 = 4x$

\# $\int dx = x + c$ $\quad$ $\int dP = P + c$ $\quad$ $\int d(x \cdot y) = xy + c$

$\int dt = t + c$ $\quad$ $\int d(MR) = MR + c$

Que- If $y = x^2 + 2$ then find integration from $x_1 = 1$ to $x_2 = 3$

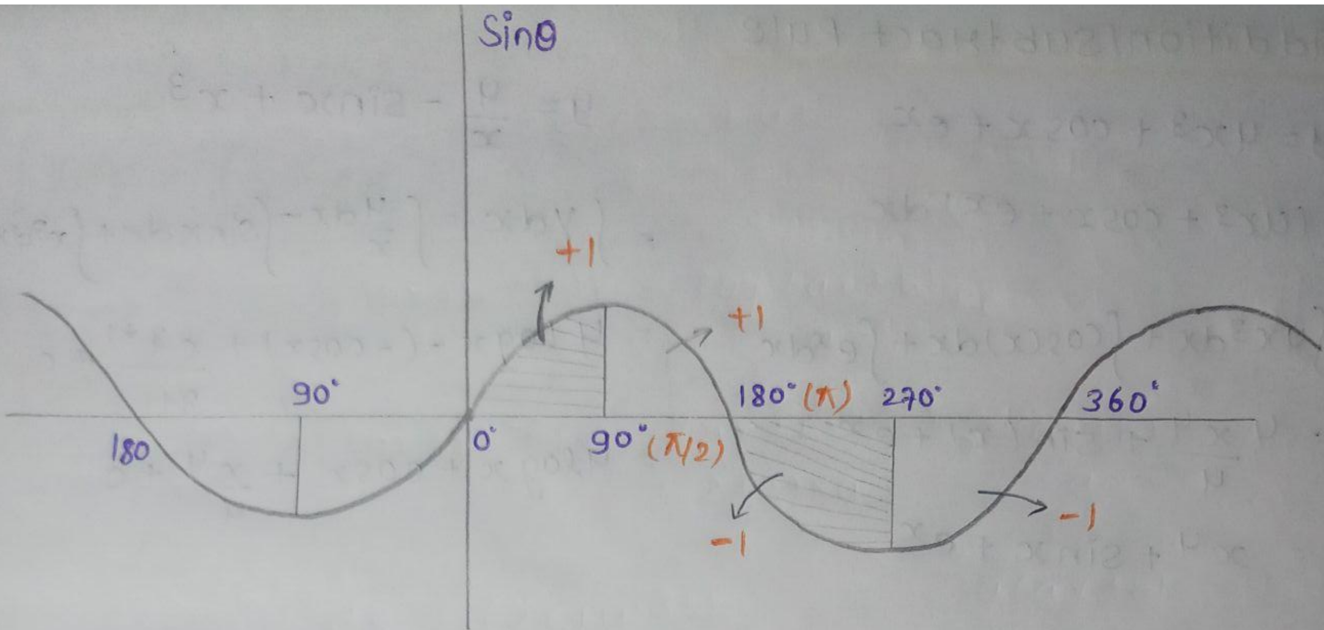
$\int y\,dx = \int_1^3 (x^2+2)\,dx = \int_1^3 x^2 dx + \int_1^3 2\,dx$

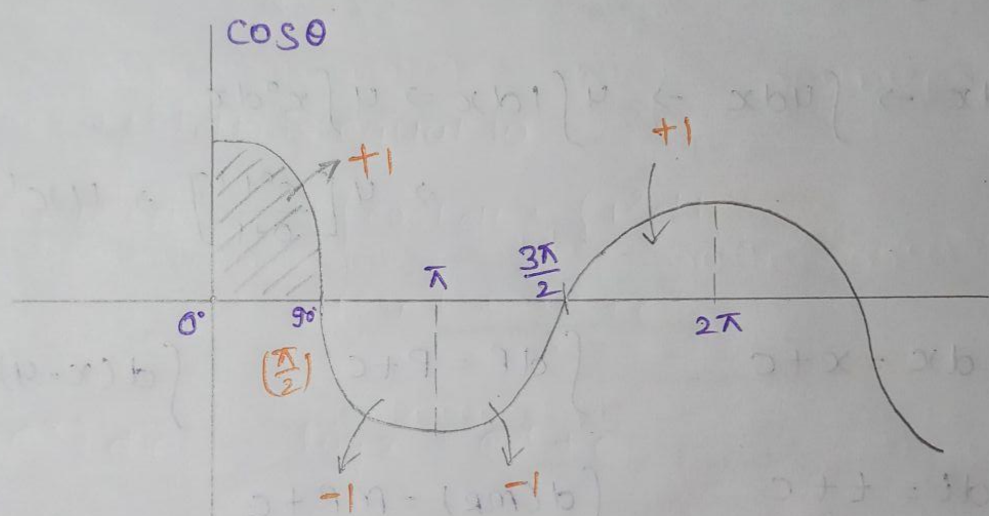
$= \left[\frac{x^3}{3}\right]_1^3 + 2(x)_1^3$

$= \frac{1}{3}[(3)^3 - (1)^3] + 2[3-1]$

$= \frac{1}{3}[27-1] + 2\times 2$

$= \frac{26}{3} + 4 \Rightarrow \frac{38}{3}$





Que- $\int_0^{\pi/2} \sin\theta \, d\theta = [-\cos\theta]_0^{\pi/2}$

$= -[\cos \pi/2 - \cos 0°]$

$= -[-1]$

$= +1$

Que- $\int_0^{\pi} \sin\theta \, d\theta = [-\cos\theta]_0^{\pi}$ $\rightarrow -[\cos\theta]_0^{\pi}$

$\rightarrow -[\cos\pi - \cos 0°]$

$\rightarrow [-1 - 1]$

$\rightarrow -[-2]$

$\rightarrow +2$

Que- $\int_0^{\pi} \cos\theta \, d\theta$

$= [\sin\theta]_0^{\pi}$

$= 0 - 0$

$= 0$

$\int_{\pi}^{2\pi} \cos\theta \, d\theta = [\sin\theta]_{\pi}^{2\pi} = 0$

$\int_0^{\pi/2} \cos\theta \, d\theta = +1$

Que- $\int_{\infty}^{r} \frac{kq_1q_2}{r^2} \, dr = kq_1q_2 \int_{\infty}^{r} \frac{1}{r^2} \, dr$

$\int \frac{1}{r^2} \, dr = \frac{r^{-2+1}}{-2+1}$

$= \frac{r^{-1}}{-1}$

$= -\frac{1}{r}$

$= kq_1q_2 \left(\frac{-1}{r}\right)_{\infty}^{r}$

$= -kq_1q_2 \left[\frac{1}{r} - \cancel{\frac{1}{\infty}}\right] 0$

$= -\frac{kq_1q_2}{r}$

Outside - Inside Rule

$\int y \, dx = \left(\frac{\text{Integration of outer function keep inside as it is}}{\text{coefficient of } x}\right)$

function of a function

\# $\int \sin(ax+b) \, dx = -\frac{\cos(ax+b)}{a}$

↳ Inside

$\int e^{(5x+2)} \, dx = \frac{e^{(5x+2)}}{5}$

↳ Inside

$\int (ax+b)^2 \, dx = \frac{(ax+b)^{2+1}}{(2+1) \times a}$

↑ Inside

Que- (i) $y = e^{5x}$

$\int e^{5x} \, dx = \frac{e^{5x}}{5} + c$

(ii) $y = \sin(2x+4)$

$$\int \sin(2x+4)\,dx = -\frac{\cos(2x+4)}{2}$$

$\int \frac{1}{(ax+b)}dx = \int (ax+b)^{-1}dx = \frac{\log(ax+b)}{a} + c$

$\int \frac{1}{(ax+b)^2}dx = \int (ax+b)^{-2}dx = \frac{(ax+b)^{-2+1}}{(-2+1)\times a} \Rightarrow \frac{(ax+b)^{-1}}{-a}$

$\Rightarrow \frac{1}{a(ax+b)}$

$y = \int (2x+4)^3 dx = \frac{(2x+4)^{3+1}}{(3+1)\times 2} + c$

Que- $\int (4\cos t + t^2)dt$ is equal to

$$\int (4\cos t + t^2)dt = 4\int \cos t\,dt + \int t^2 dt$$

$$= 4\sin t + \frac{t^3}{3} + c$$

Que- $\int \left(\frac{1}{x} + e^x\right)dx = \log x + e^x + c$

Que- $y = 2x+3$, find integer from $x=1$ to $x=2$

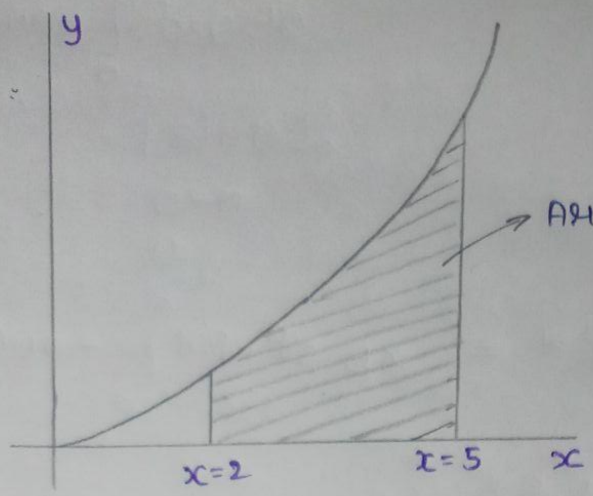
$$\int y\,dx = \int_1^2 (2x+3)\,dx = 2\int_1^2 x\,dx + \int_1^2 3\,dx$$

$$= \left(\frac{2x^2}{2}\right)_1^2 + 3(x)_1^2$$

$$= [(2)^2 - (1)^2] + 3(2-1)$$

$$= 3 + 3(1) = 6$$

Que- Find area of curve $y = x^2$ from $x_i = 2$ to $x_f = 5$



$$\text{Area} = \int V dx = \int x^2 dx = \left(\frac{x^3}{3}\right)_2^5$$

$$= \frac{1}{3}\left[(5)^3 - (2)^3\right]$$

$$= \frac{117}{3} = 39$$

Average Velocity

$$\langle \boxed{V} \rangle_{Avg} = \frac{\int \boxed{V} dt}{\int dt}$$

Average acceleration

$$\langle accn \rangle = \frac{\int a dt}{\int dt}$$

Average momentum

$$\langle P \rangle = \frac{\int P dt}{\int dt}$$

Que- If velocity $V = 2t+1$, then find Avg. velocity in 2-sec

$$V_{Avg\ time} = \frac{\int V dt}{\int dt} = \frac{\int_0^2 (2t+1) dt}{\int_0^2 dt} \Rightarrow \frac{\left(\frac{2t^2}{2}\right) + (t)_0^2}{(t)_0^2}$$

$$\Rightarrow \frac{(2)^2 + (2-0)}{2-0}$$

$$\Rightarrow \frac{4+2}{2} = \frac{6}{2} \Rightarrow 3 m/sec$$

At 2 Sec → Instant

In 2 Sec → Intraval (0-2 Sec)